While I have wished to place on record the matter herein before dealt with, I am perfectly aware of its drawbacks—in point, for example, of friction, flexure, and the difficulty of permanently securing the hands. I noticed on my last visit that while the clock itself was "on time," the *minute* hands were very erratic. This, I am told, is largely due to the fact that in their repairs to the Tower, long subsequent to my restoration of the clock, the masons shaped their stones, in the lantern, on *carpet* above the bell frame and just over the tube leading to the South dial. This said carpet got wound round, or entangled with the tube and had to be cut up to get it out, causing the whole gear to be much shaken up.

I see by my correspondence that in the year 1921 I suggested new dials, but the authorities did not see their way, at the time, to carry this out. I again urge this—of skeleton, and, if possible, of *rhomboidal* form, with new balanced hands and *motion-work*. But, let the existing motion-work be mounted somewhere in the Clock Room with a suitable tablet underneath it.

That I found my friend will be manifest from the foregoing. And "here's to our next happy meeting." I look back to these Colyton days as amongst the happiest of my long horological experiences.

The photograph is by kind permission of the artist, Mr. A. Hartley, of Colyton.

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

I.—Representations of Vibrations by Vectors.

1. *Introduction.*

EVER since the pendulum was first applied to clocks by Christian Huygens in 1656, it has been the subject of much mathematical research. Yet even now, more than a century after Airy's classical paper,* there is a lack of exact knowledge as to the rate errors produced by the various disturbing causes and no accepted answer to such questions as:—

(*a*) At what amplitude should the pendulum be run?

(*b*) At what point of the vibration should the sustaining forces be applied?

(*c*) Should the pendulum be run in an atmosphere of air or of another gas?

(*d*) At what pressure should this atmosphere be?

(*e*) Should the suspension spring be long or short, thick or thin?

It is the aim of this paper to give a full discussion of the essential causes of variations in the rate of a pendulum and to show the way in which answers to the above questions should be obtained; we shall see that some of the answers vary with the design of the pendulum and with the type of escapement employed.

Afterwards we shall apply the principles enunciated to certain typical escapements and consider the ultimate accuracy which it may be possible to attain.

For the most part, the discussion will refer directly to the ordinary pendulum, but the main principles apply equally to balances, and to torsional pendulums; in fact, by a suitable change in the meaning of the symbols, they apply to sustained vibrations of any kind whatever, electrical as well as mechanical.

The theory will be studied with the aid of rotating vectors, a method which was introduced by Blakesley in 1885 in connection with alternating currents of electricity. It has proved of immense service in that connection by bringing the theory within the grasp of students whose mathematical attainments are insufficient for a purely analytical method, and in helping one to visualise the phase relationships between the various actions. Hitherto, the method has received scant attention from writers on the pendulum, but the author has found it of great value here, too.

Before proceeding with our subject proper, it is necessary to give a few definitions and a short explanation of the vector method.

* G. B. Airy. "On the Disturbances of Pendulums and Balances, and on the Theory of Escapements." Cambridge Phil. Soc. Trans. Vol. III., part 1, pp. 105-128. (n.d., but paper read on 27th November, 1826.)

2. Elements of a Vibration.

A vibration may be defined as a change of some kind which takes place continuously on opposite sides of some mean state; with a pendulum, or any other mechanical vibration, we have such changes in the displacement from the mean position, in the velocity, and in the acceleration. With electrical oscillations the corresponding elements are the charge in a condenser, the current in the circuit, and the rate of change of that current.

We shall restrict our attention to those vibrations in which the changes on the negative side of the mean are an exact replica of those on the positive side, and in which the same set of changes is repeated over and over again, always in the same time. The greatest value on either side of the mean is conveniently referred to as the "crest value." It is commonly termed the "amplitude," but we shall find it desirable to apply that name to simple harmonic vibrations only, and to give it a slightly different meaning in other cases.

The complete set of changes (that is *both* the positive and negative ones) which is continually being repeated is the "cycle," the time taken to perform one cycle is the "period," and the number of cycles performed per unit of time is the "frequency." It should be noted that it is customary to specify a pendulum by its time of swing, which is half the period. Thus a "seconds" pendulum has a period of two seconds.

The particular point of its cycle at which the vibration happens to be at some chosen instant of time is its "phase" at that instant. Two vibrations are in phase with one another when they are at corresponding points on their cycles (say the positive crest) at the same instant; otherwise they are "out of phase" with one "ahead" or "leading" and the other, "behind" or "lagging."

The case in which there is a quarter cycle phase difference is so very important that it requires a special name; the two vibrations are then "in quadrature." For the simplest types of vibration, at least, one passes through its crest at the same instant as the other passes through zero when they are in quadrature.

3. Properties of Rotating Vectors.

It is convenient to represent the flux of time by the rotation of a line about one end, and the changing value of the displacement, or other quantity under consideration, by the projection of that line on a fixed reference line, using some suitable scale.

The revolving line is termed a "rotating vector," or simply a "vector," and its direction, specified by the complement of the angle between it and the reference line, gives the phase of the vibration at the instant for which the diagram is drawn. Obviously, the vector must go once round for one complete cycle; hence the time of rotation of the vector is equal to the period of the vibration, or the frequency of rotation is the same as the frequency of the vibration represented.

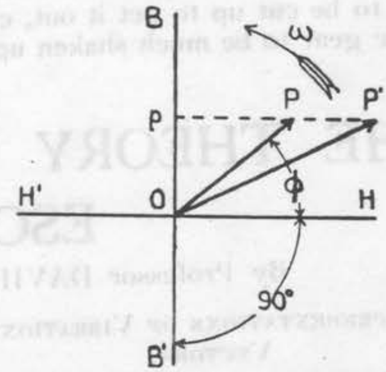


FIG. 1.—*Vectors and Projection.*

In Fig. 1, OP is such a rotating vector and Op is its projection on the reference line BB′; ϕ is its phase at the instant considered, and ω is its "phase velocity," that being the angular velocity of the line OP. The relations between the quantities are:—

$\phi=\phi_0+\quad \omega dt=\phi_0+\omega t$ if ω be constant (3.01)

$Op=OP\times\sin\phi=OP\times\sin(\omega t+\phi_0$ if ω be constant . (3.02)

The mean phase velocity for the whole cycle is:—

$\omega=2\eta/T_0=2\eta f$. (3.03)

where T_0 is the period and f the frequency of the vibration.

Unless otherwise stated, it is to be assumed that the time is counted from the instant at which the displacement is zero, so that $\phi_0=0$ for the displacement vector.

To avoid continual repetition, we shall refer to the "projection of a vector on the reference line" simply as its "projection," and to all lines parallel to the reference line as "vertical," although in fact BB′ might be drawn in any direction. Lines such as OH which are at right angles to BB′ are "horizontal." The length of the vector at any instant is the "amplitude" at that instant.

Observe that the direction of the vector has nothing whatever to do with the direction of the quantity represented if that quantity be a directive one; the vector direction is used to represent phase, and changes continuously as time progresses.

Any other vector through O, such as OP′, would give the same projection as OP if its outer end be at the same level as p and P. Consequently, the vector to represent a given quantity may be drawn in any direction provided it has the correct length; or of any length not less than the projection itself, if it be drawn in a suitable direction. There is, however, only one position at each instant which would be consistent with some specified manner of rotation, such as uniform rotation, as it is desirable to make the vector pass through the horizontal at the instant when the quantity represented passes through zero so as to avoid vectors of infinite length. Uniform rotation is generally the most convenient, and is to be assumed if nothing else is stipulated.

Changes in Op can be brought about by the rotation of OP, and also by changes in the length of OP, or by both. By suitable changes in the amplitude or phase velocity, or both, p can be made to perform any kind of vibration whatever.

4. *Vector Addition.*

When a number of vectors are drawn so that each one begins where the previous one ends (see Fig. 2), we get an open polygon with the arrows pointing the same way round it. Or the closing side of the polygon, taken the other way round, is termed the "resultant" of the given vectors, which are themselves the "components" of the resultant.

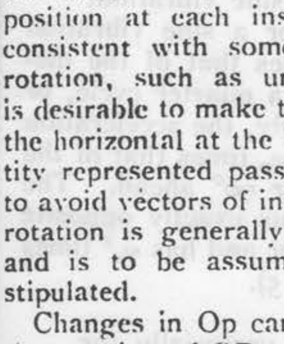


Fig. 2.—*Vector Addition.*

Now, the projections of equal parallel lines are themselves equal; consequently the projections of our component vectors are not altered in length by the fact that the vectors are drawn to form a polygon instead of radiating from a point. Hence it is obvious from the diagram that:—

Projection of resultant = algebraic sum of the projections of the components.

In other words, if the individual amplitudes be X_1, X_2, X_3, etc., and their projections x_1, x_2, x_3, etc., and if

$X = X_1 + X_2 + X_3 +$, etc., be true *vectorially* for the *amplitudes* . (4.01)

then

$x = x_1 + x_2 + x_3 +$, etc., must also be true *algebraically* for the *projections* (4.02)

This gives the first law upon which the vector method of studying vibrations is based, namely, that any equation which is true vectorially (that is by the polygon of vectors) for the amplitudes must also be true algebraically for the instantaneous values represented by the projections. This law holds true when the vectors rotate, even when the polygon changes its shape owing to differences in the phase velocities of the several vectors.

The converse statement is commonly true, but not always. Thus, in Fig. 3, where R is taken at the same level as the end of the last vector of the polygon, equation 4.02 is true but 4.01 is not.

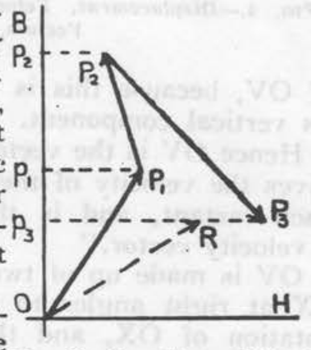


Fig. 3.—*Equation not true Vectorially.*

When the vectors revolve at the same rate and keep constant proportions between their lengths, if R and P_3 do not coincide they will only be at the same level at two points on each revolution; consequently in such cases the vector equation must hold if the algebraic one be true for the instantaneous values at *every* instant. In other cases it is necessary to scrutinise the conditions before this statement can be made.

5. *Displacement, Velocity and Acceleration Vectors.*

In Fig. 4, let OX be the displacement vector, so that the displacement from the mean position at the instant to which the diagram refers is Ox.

At this instant, the point X on the diagram is moving in a direction parallel to OV at a rate represented by the length of that line. Obviously any horizontal motion of X cannot affect the position of x, and the velocity of the latter point (which gives the rate of change of the quantity represented by Ox) must be the same as the vertical motion of X. Thus, the velocity of the vibrating point is given by the projection

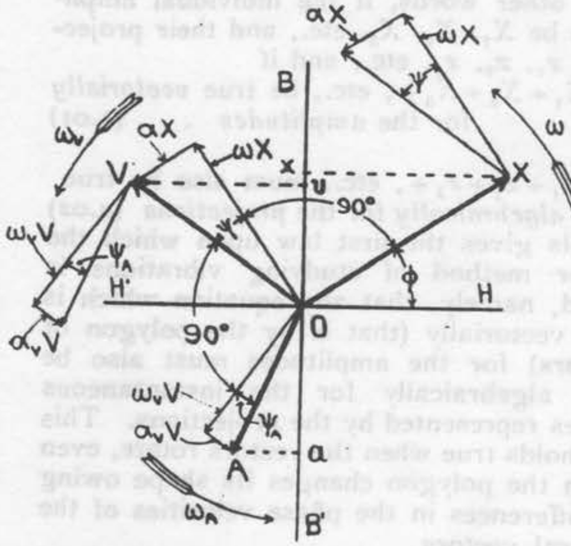


FIG. 4.—*Displacement, Velocity and Acceleration Vectors.*

of OV, because this is the same thing as its vertical component.

Hence OV is the vector whose projection gives the velocity of the vibrating point at each instant, and is therefore called the "velocity vector."

OV is made up of two components; one ωX at right angles to OX is due to the rotation of OX, and the other αX along XO is caused by its shrinkage, α being the fractional rate of decay of the amplitude OX. Thus, the velocity amplitude is:—

$$V = \sqrt{\omega^2 + \alpha^2}\, X \qquad (5.01)$$

and it lies ahead of OX by the angle $(90° + \psi)$, where

$$\tan \psi = \alpha/\omega \qquad (5.02)$$

We thus have the second law:—

The rate of change of a quantity which is represented by the projection of a vector whose phase velocity is ω and whose fractional rate of decay is α, can be represented by another vector of $\sqrt{\omega^2 + \alpha^2}$ times the amplitude and $(90° + \psi)$ ahead in phase, where $\tan \psi = \alpha/\omega$.

The rate of change of velocity is the acceleration, and can be represented by another vector in the same way; but the α and the ω are then the values applicable to the velocity vector, which would not always be the same as for the displacement vector.

The simplest case is, of course, that in **which the vectors rotate at constant velocity and retain constant lengths, in which case** α **and** ψ **are both zero.** This is the simple harmonic motion, or sine vibration.

The next is that in which α **is constant,** but not zero, as well as ω. This is the "shrinking sine function," which the author has previously treated rather fully with special reference to electrical oscillations.* In each of these cases, the three vectors (displacement, velocity and acceleration) have the same constant phase velocity, and A/V is the same as V/X.

6. *Simple Harmonic Vibration.*

When the vector has a constant length and its phase velocity has a constant value ω_0, the projection of its end performs a simple harmonic motion or sine vibration.

The velocity vector for a sine vibration has an amplitude ω_0 times that of the displacement vector and is a quarter cycle, or 90° ahead of it. Similarly, the acceleration vector has an amplitude ω_0 times that of the velocity and is a further 90° ahead. The acceleration vector is thus exactly opposite to the displacement vector and has ω_0^2 times the amplitude (see Fig. 5).

In other words,

$$\omega_0^2 X + A = 0 \text{ is true vectorially for the amplitudes} \qquad (5.01)$$

and consequently,

$$\omega_0^2 x + x = 0 \text{ is true algebraically for the instantaneous values} \qquad (5.02)$$

where x is the acceleration at the instant considered.

This is the characteristic differential equation for the simple harmonic vibration.

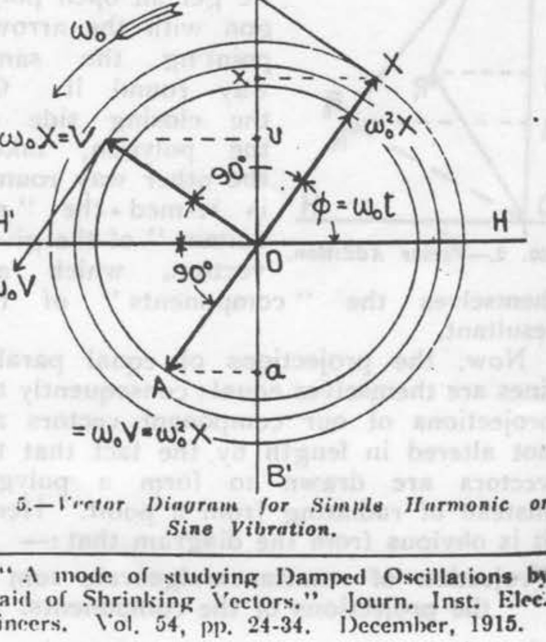


FIG. 5.—*Vector Diagram for Simple Harmonic or Sine Vibration.*

* "A mode of studying Damped Oscillations by the aid of Shrinking Vectors." Journ. Inst. Elec. Engineers. Vol. 54, pp. 24-34. December, 1915.

Conversely, when such an equation is obtained from the conditions of the motion of a given body, we know that the vibration is a simple harmonic one.

The following equations are of importance:—

$$\text{Since } \ddot{x} = -\omega_0^2 x \quad (5.03)$$

the phase velocity is:—

$$\omega_0^2 = \sqrt{-\ddot{x}/x} = \sqrt{\text{acceleration (reversed)}} \div \text{displacement} \quad (5.04)$$

The restoring force is:—

$$F = -M\ddot{x} = (M\omega_0^2)\, x = kx \quad (5.05)$$

where k is the restoring constant, or

$$k = M\omega_0^2 = F/x = \text{restoring force per unit displacement} \quad (5.06)$$

The equation for the phase velocity may also be written:—

$$\omega_0 = \sqrt{k/M}$$
$$= \sqrt{\text{controlling constant} \div \text{inertia}} \quad (5.07)$$

The period is:—

$$T_0 = 2\eta/\omega_0 = 2\eta\sqrt{M/k} \quad (5.08)$$

and the frequency,

$$f = 1/T_0 = \omega_0/2\eta = (\sqrt{k/M}) \div 2\eta \quad (5.09)$$

It should be particularly noted that the amplitude does not appear in the expression for the period; in other words, the period of a simple harmonic vibration is the same whether the amplitude be small or great.

A certain amount of energy is associated with the vibration and oscillates between the potential and kinetic forms. At the ends of the swing where the velocity is zero, the energy is wholly potential, while at the centre of the swing it is wholly kinetic.

We shall see later that it is important to remember that an action which increases the kinetic energy of the vibrating body does not necessarily increase the total energy of the vibration.

At any other instant, at which the phase is ϕ, the potential energy is:—

$$W_p = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kX^2 \sin^2\phi$$
$$= \tfrac{1}{2}M\omega_0^2 X^2 \sin^2\phi \quad (5.10)$$

the kinetic energy is:—

$$W_k = \tfrac{1}{2}Mv^2 = \tfrac{1}{2}M\omega_0^2 X^2 \cos^2\phi$$
$$= \tfrac{1}{2}kX^2 \cos^2\phi \quad (5.11)$$

and the total energy of the vibration is:—

$$W_T = \tfrac{1}{2}kX^2 (\sin^2\phi + \cos^2\phi)$$
$$= \tfrac{1}{2}kX^2 = \tfrac{1}{2}MV^2 \quad (5.12)$$

since $\sin^2\phi + \cos^2\phi = 1$.

Equation 5.05 shows us that the essential thing for a true sine vibration is that the restoring force should be exactly proportional to the displacement, or that the ratio (restoring force $\div$ displacement) be exactly constant.

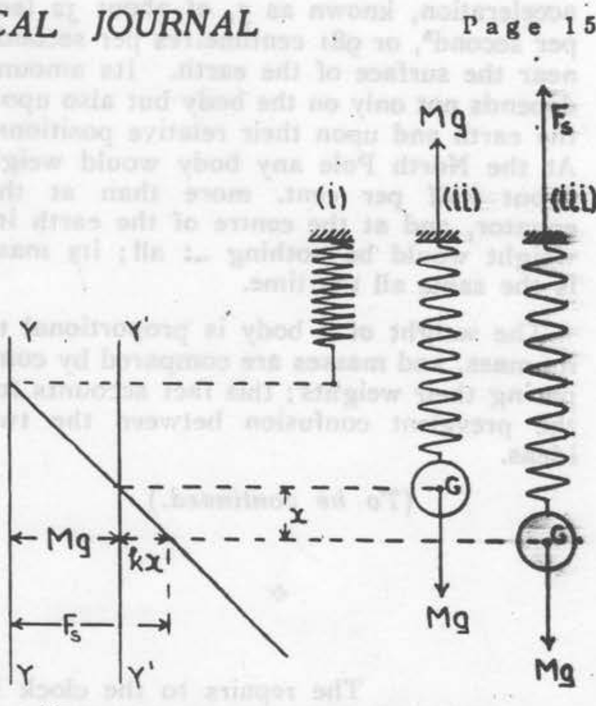


Fig. 6.—*Body with Harmonic Vibration.*

(i) Spring free.
(ii) Spring loaded and at rest.
(iii) Spring stretched beyond equilibrium point.

If a mass be suspended by a helical spring it will come to rest at that position in which the tension of the spring just balances the weight of the mass. When the mass is displaced from that position, the pull of the spring will increase for downward movements or decrease for upward ones. The change will be proportional to the displacement if the spring be an ideally perfect one, and it is a restoring force tending to return the mass to its original position.

If let go, the mass will then perform a true sine vibration, at least if the friction and the inertia and weight of the spring can be neglected.

At this point it is desirable to emphasise the necessity of having clear ideas as to mass and weight. The mass of a body is a property of its own; it is a fundamental idea which cannot be defined any more than time or distance, but must be learnt by experience like these ideas. It is sometimes called "quantity of matter," but that is only an alternative name, not a definition; it does perhaps help one to grasp the idea.

On the other hand, the weight of a body is the force with which the earth attracts it, and causes it to fall towards the earth, when acceleration, known as *g*, of about 32 feet per second², or 981 centimetres per second² near the surface of the earth. Its amount depends not only on the body but also upon the earth and upon their relative positions. At the North Pole any body would weigh about half per cent. more than at the equator, and at the centre of the earth its weight would be nothing at all; its mass is the same all the time.

The weight of a body is proportional to its mass, and masses are compared by comparing their weights; this fact accounts for the prevalent confusion between the two ideas.

(*To be continued.*)

❖

St. George's Clock. The repairs to the clock in the tower of St. George's Church, Gravesend, are apparently going to cost more than anticipated, and there would appear to be no immediate prospect of matters being put right. The original St. George's clock dates back nearly two hundred years, and there would therefore seem to be no reason to complain if repairs have become essential. The question will arise, we presume, whether in the end it will not be cheaper to place a new clock in the tower.

British Industries Fair.

ASSOCIATIONS in the jewellery and watch and clockmaking trades have been asked by Mr. Douglas Hacking, M.P., Parliamentary Secretary to the Department of Overseas Trade, to co-operate in connection with the British Industries Fair which is to be held simultaneously in London and Birmingham next year from February 18th to March 1st.

"Thanks largely to the help which we received from trade Associations and from other representative bodies, the Fair held last February," writes Mr. Hacking, "achieved a decisive success, the number of exhibitors, the exhibiting area occupied, and the attendance of buyers from overseas and home, surpassing all previous figures.

"I confidently expect that the attendance of buyers from overseas and home will exceed the record figures of last year, and you will agree with me that it is of great importance that these visitors should find the display thoroughly representative of the best that Britain can produce.

"The growth in size, prestige and goodwill of this National Fair is contributing year by year to the gradual recovery by this country of its former prosperity. It can, however, continue to play its part in the country's welfare only so long as it enjoys the support of the representative industrial and commercial organisations of the United Kingdom."

FITTING A CONCEALED KEYLESS WINDER TO A KEY-WOUND WATCH.

By a Leicester Member of the B.H.I.

THERE are probably hundreds of good and serviceable high-class English-made watches, all capable of good performances, which have fallen into disuse solely because of the unfortunate circumstance that they have to be wound by a separate key.

I have heard of a batch of key-wound but otherwise first-class watch movements which have never even been cased, and never will be.

I have been the possessor of such an unfortunate key-wound watch. When it came into my possession years ago it was not such a "crime" for a watch to be key-wound. It has, however, always been an excellent timekeeper, and has had such low position errors that I continued to wear it after "keyless" became the vogue.

Several years ago I devised and made for it a key screwed on to the fusee stem after the manner of an alarm clock key, but shallow enough to enable the case to be shut. Like the alarm clock key, however, it needed re-grasping with finger and thumb after each half-turn, and while winding (particularly in the dark) I often wished I could retain my grasp and wind

Examples of the Ingersoll Watch from 1891 to 1928. The original watch was 2¼in. in diameter and 1¾in. thick. The present-day model is 1 15/16in. in diameter and ⅜in. thick.

"the watch that made the dollar famous." In 1908 the Ingersolls purchased the factory and business of the Trenton Watch Company, of Trenton, New Jersey, and began watch manufacturing on their own account. In 1914 they purchased the plant of the New England Watch Company, formerly the Waterbury Watch Company, of Waterbury, Connecticut.

In its first year 12,000 Ingersoll watches were made. To-day the output is 25,000 a day.

It is a matter for regret that Robert H. Ingersoll had for the last thirty years of his life suffered from bad health. At the time of his death he was no longer connected with the business, having sold out his interest in 1922.

A British Company was formed in 1916, the Managing Director of which is Mr. E. S. Daniells, who started the Ingersoll business in this country in 1905 and has been connected with the firm for forty years. Through the courtesy of the Ingersoll Company we are able to give an illustration of a range of Ingersoll watches showing the evolution of the watch from 1891 to the present time.

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

(Continued from page 16).

II.—The Pendulum.

7. *The Simple Pendulum.*

A simple pendulum consists of a small bob, of negligible dimensions and mass M, suspended from a fixed point O by a perfectly flexible string whose mass and weight can be neglected.

When the bob is displaced from one side and then let go, it oscillates to and fro along the arc EE′, about the mean position C (Fig. 7).

When the bob is at M, its displacement from C, measured along the arc is x. Its weight, Mg, may be resolved into components along the radius and the tangent to the arc at M. The radial component is balanced by the tension of the string, but the tangential one constitutes a controlling force tending to return the bob to its central position and thus causes an acceleration towards C. The amount of this restoring force is:—

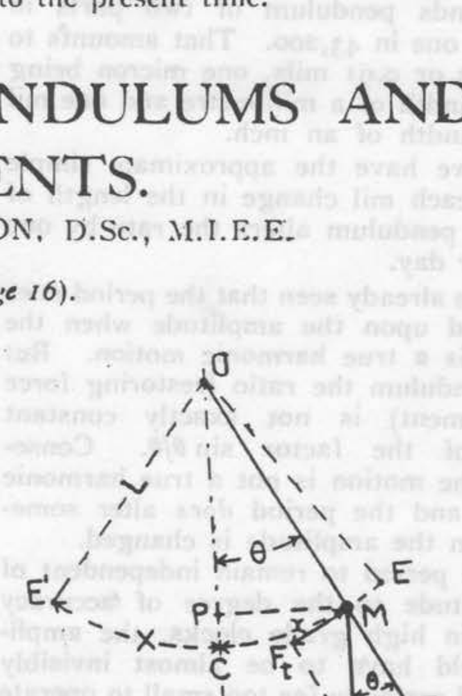


Fig. 7.—*The Simple Pendulum.*

$$F = Mg \sin \theta = Mg\ MP/OM$$
$$= (Mgx/L)\ (\sin \theta/\theta) \qquad (7.01)$$

since

$$MP : MC = L \sin \theta : L\theta = \sin \theta : \theta \qquad (7.02)$$

Now, if the displacement be very small compared with the radius L, we may neglect the difference between the semi-chord MP and the arc MC, or between $\sin\theta$ and θ, and then we may write:—

$$F = Mg\,x/L = (Mg/L)\,x$$
$$= \text{constant} \times \text{displacement} \qquad (7.03)$$

Consequently, *so long as the maximum displacement is small enough to make the above approximation sufficiently accurate* for our purpose, the bob performs a simple harmonic vibration whose controlling constant is:—

$$\kappa = F/x = Mg/L \qquad (7.04)$$

and whose period is:—

$$T_0 = 2\pi\sqrt{(M/k)} = 2\pi\sqrt{(L/g)} \qquad (7.05)$$

At Greenwich, where $g = 981.18$ centimetres per square second, the length of the seconds pendulum is 994.2 mm., or 39.14 inches.

When dealing with small changes in L/g, etc., it should be remembered that $\sqrt{(1+n)} = (1+\frac{1}{2}n)$ when n is small. Now, there are 86,400 seconds in a mean solar day, or a million seconds in 11.574 days. Hence, to change the rate by one second per day requires an alteration in the length of a seconds pendulum of two parts in 86,400, or one in 43,200. That amounts to 23 microns or 0.91 mils, one micron being one thousandth of a millimetre and one mil the thousandth of an inch.

Hence we have the approximate simple rule that each mil change in the length of a seconds pendulum alters the rate by one second per day.

We have already seen that the period does not depend upon the amplitude when the vibration is a true harmonic motion. But with a pendulum the ratio (restoring force ÷ displacement) is not exactly constant because of the factor $\sin\theta/\theta$. Consequently, the motion is not a true harmonic vibration and the period *does* alter somewhat when the amplitude is changed.

For the period to remain independent of the amplitude to the degree of accuracy required in high grade clocks, the amplitude would have to be almost invisibly small, and certainly far too small to operate any known form of escapement.

Any vibration, such as that of a pendulum, which is very nearly, but not quite, a true harmonic one may be termed a "disturbed harmonic vibration."

When the conditions of a vibration differ from some ideal one, owing to the presence of some disturbing factor, the period will usually be different from that of the ideal vibrator. The fractional excess of rate due to that factor is its "deviation."

Should the disturbing factor change erratically, so as to alter the rate, the increase of deviation due to the accidental change is the corresponding "error."

Thus, if Δ be the deviation, and δ the error, we have:—

$$\Delta = (T_0 - T)/T_0 \qquad (7.06)$$
$$\delta = \Delta_2 - \Delta_1 \qquad (7.07)$$

Negative values of Δ or δ correspond to decreases of rate or increases of period.

It is very important to distinguish clearly between the two quantities deviation and error, because the single term "error" is commonly employed for both, and the result is much confused thinking and many erroneous conclusions. The deviations are allowed for unconsciously when the pendulum is adjusted by trial in the ordinary way, and they are of no importance whatever so long as they remain constant. The errors are the erratic alterations in the deviations due to accidental causes more or less beyond control, and cause irregularities in the time-keeping. Too often, people have sought to get small or zero deviations, when the real thing that is wanted is a minimum value for the errors.

The particular deviation due to the factor $\sin\theta/\theta$ is the "circular deviation," and the corresponding error the "circular error." We shall see later that the former is:—

$$\Delta_c = -\Theta^2/16 = -4.57 \text{ secs. per day} \times (\Theta/100 \text{ arc-minutes})^2 \qquad (7.08)$$

where Θ is the amplitude.

8. *The Actual Pendulum.*

With a real pendulum, the motion is an angular one about an axis passing through the mid-point in the length of the suspension spring, and the mass is not concentrated at a single point but is distributed throughout an appreciable volume.

As a result of these facts, different portions of the mass are moving at different velocities and the effective inertia is greater than if the whole mass were collected at the centre of gravity.

The angular inertia is what is called the "moment of inertia," which we may regard as the angular equivalent of mass. To get it, we must divide the whole volume into a very large number of exceedingly small parts, multiply the mass of each part by the square of its distance from the axis vibration, and add all these products ;ether.

The shorthand way of expressing this is write:—

Moment of inertia $= K = \Sigma (M_1 L_1^2)$ (8.01)

…ich simply means what we have written …t in full above.

With small amplitudes, the motion is still very nearly a harmonic vibration; instead of writing out a fresh set of equations, we shall make the same ones do, but when we are dealing with angular vibrations instead of linear ones, we must interpret the symbols in our previous equations as follows:—

x, X = angular displacement and amplitude, expressed in radians. (1 radian $= 57.296° = 3{,}437.7$ arc-minutes.)

v, V = angular velocity.

F = angular force, or torque, τ.

k = restoring torque per unit angular displacement.

M = angular mass = moment of inertia $= K$.

If M be the total mass of the pendulum and L the distance of its centre of gravity from the axis of vibration, we have:—

The restoring torque with an angular displacement θ is:—

$$\tau = MgL \sin\theta \quad (8.02)$$

$$= \Sigma (M_1 L_1) g \, . \sin\theta \text{ if there are several parts} \quad (8.03)$$

Controlling constant,

$$k = \tau/\theta = MgL (\sin\theta)/\theta \quad (8.04)$$

$$\doteq MgL \text{ when } \theta \text{ is very small} \quad (8.05)$$

$$= \Sigma (M_1 L_1) g \text{ if there are several parts} \quad (8.06)$$

Inserting these values in the formula for the period, we get:—

$$T = 2\pi \sqrt{(\text{inertia} \div \text{controlling constant})} \quad (8.07)$$

$$= 2\pi \left[\Sigma (M_1 L_1^2) \div \Sigma (M_1 L_1) g\right]^{\frac{1}{2}} \quad (8.08)$$

$$= 2\pi \left[\{ \Sigma (M_1 L_1^2) \div \Sigma (M_1 L_1) \} \div g\right]^{\frac{1}{2}} \quad (8.09)$$

Comparing this result with equation …5, we see that the length of the simple …ndulum with the same period is:

$$L_0 = \Sigma (M_1 L_1^2) \div \Sigma (M_1 L_1) \quad (8.10)$$

When making such calculations for a …dulum whose drawings are given, the …ss of each part can be estimated from … dimensions and the density of the …terial of which it is made. The position … the centre of gravity can also be cal…lated from the drawings, and the other …mensions scaled off.

With parts of appreciable size, such as the bob and the rod, the moment of inertia exceeds the product of the mass and the square of the distance of its centre of gravity from the axis of vibration (which, it should be remembered passes through the centre of the length of the suspension spring, not the top) by an amount equal to its moment of inertia about a parallel axis through its centre of gravity.

Thus, for cylindrical bob of length l and diameter d,

$$K_{bob} = M_{bob} (L^2_{bob} + l^2/12 + d^2/16) \quad (8.11)$$

and for a rod whose lower end is distant l_1 and upper end l_2 from the axis of vibration,

$$K_{rod} = M_{rod} (l_1^2 + l_1 l_2 + l_2^2) \div 3 \quad (8.12)$$

$$\doteq M_{rod}\, l_1 (l_1 + l_2) \div 3 \, . \quad (8.13)$$

The other parts in the usual design of pendulum are of sufficiently small dimensions for it to be assumed that each is collected at its centre of gravity, except perhaps the crutch when there is one.

The work should be done in tabular form, as in the example shown in Table I., which refers to a standard commercial pendulum made by Messrs. Gent and Co., Ltd., of Leicester. With the setting of the bob used in the calculation, the equivalent length is somewhat longer than that of a seconds pendulum, but when allowance is made for the suspension spring, etc., it will be found that the setting is not far wrong.

It will be observed that the equivalent simple pendulum is very little different from that which would be obtained by supposing the whole mass to be collected at the c.g. of the bob, and this statement will be found to apply to almost any common design of clock pendulum. It is therefore convenient to put the equation into a form which shows at once how the various items cause the period to deviate from that of such a simple pendulum.

Let the mass of the bob alone be M, the distance of its c.g. from the axis of vibration be L, and its radius of gyration about a parallel axis through its c.g. be r. Let M_1, M_2, etc., L_1, L_2, etc., r_1, r_2, etc., be the corresponding quantities for the other parts of the pendulum, including the crutch when there is one. Further, let M_T, L_T, r_T, refer to the pendulum as a whole. Then,

$$\Sigma (M_1 L_1) = ML + M_1 L_1 + M_2 L_2 + \text{etc.} = M_T L_T \quad (8.14)$$

$$= ML \{ 1 + M_1 L_1/ML + M_2 L_2/ML + \text{etc.} \} \quad (8.15)$$

TABLE I.

CALCULATION OF EQUIVALENT SIMPLE PENDULUM FOR PENDULUM NO. 61.

Part.	Mass M Grams.	L mm.	Moment about O. ML 10^3 gram.-mm.	2nd Moment about O.	L^2 etc. 10^3 mm².	ML^2 etc. 10^6 gram-mm².
1	2	3	4	5	6	7
Bob (Steel)	4590	1004	4608	L^2	1008	4627
10″ × 2¼″ diam.				$l^2/12$	5.4	25
254 mm. × 57 mm. ...				$d^2/16$	0.27	1
Rod (Invar)	340	560	190	$l_1{}^2/3$	410	140
42¾″ × ½″ × ⅛″ + head ...				$l_1 l_2/3$	4	1
$l_1 = 1110$ mm., $l_2 = 10$ mm.						
Screw (Steel)	30	1130	34		1280	38
3″ × $\frac{5}{16}$″ diam.						
Rating Nut, etc.	110	1150	127		1320	145
Totals	5070		4959			4977

Distance of c.g. below axis of vibration. $L_T = \Sigma\,(ML) \div \Sigma M = 978$ mm.

(Radius of gyration)² about do. $(L_T{}^2 + r_T{}^2) = \Sigma\,(ML^2) \div \Sigma M = 981 \times 10^3$ mm.²

Radius of gyration do. do. $(L_T{}^2 + r_T{}^2)^{\frac{1}{2}} = 990$ mm.

Length of equivalent Simple Pendulum. $L_0 = \Sigma\,(ML^2) \div \Sigma\,(ML) = 1003.5$ mm.

Suspension Spring. $l = 0.70$ inch $= 18$ mm.

$b = 0.51$ inch $= 13$ mm.

$c = 16$ mils $= 0.41$ mm.

$$\Sigma\,(M_1 L_1{}^2) = M\,(L^2 + r^2) + M_1\,(L_1{}^2 + r_1{}^2) + \text{etc.} \qquad (8.16)$$

$$= ML^2\,\{\,1 + r^2/L^2 + (M_1/M)\,(L_1{}^2 + r_1{}^2)/L^2 + \text{etc.}\,\} \qquad . \qquad . \qquad (8.17)$$

Hence,

$$T_0 = 2\pi\,(L/g)^{\frac{1}{2}}\,\frac{\{\,1 + r^2/L^2 + (M_1/M)\,(L_1{}^2 + r_1{}^2)/L^2 + \text{etc.}\,\}^{\frac{1}{2}}}{\{\,1 + M_1 L_1/ML + M_2 L_2/ML + \text{etc.}\,\}^{\frac{1}{2}}} \qquad (8.18)$$

$$\doteq 2\pi\,(L/g)^{\frac{1}{2}}\,[\,1 + \underbrace{\tfrac{1}{2}r^2/L^2}_{\text{Dimensions of bob.}} + \underbrace{\tfrac{1}{2}\,(M_1/M_2)\,(L_1{}^2/L^2 + r_1{}^2/L^2 - L_1/L)}_{\text{Rod.}} + \underbrace{\text{etc.}}_{\text{Other parts.}}\,] \qquad . \qquad . \qquad (8.19)$$

This equation is not quite complete, for we still have to allow for the stiffness of the suspension spring, for the buoyancy and inertia of the air, and for the deviations caused by the amplitude and by the frictional and driving forces, and perhaps also for some flexibility in the supports.

Each of these items will add an extra term inside the large brackets of equation 8.19, and will have to be discussed separately as we proceed.

The following errors in the first article should be corrected :—

p. 11, Footnote.—" 1926 " should read " 1826."

p. 13, 18th line from bottom.—" Or the closing side " should read " Or, the closing side."

The equation numbers in section 6 should begin with 6, not 5, and the following equations should appear as here given :—

$$\omega = 2\pi/T_0 = 2\pi f \qquad . \qquad 3.03$$

$$V = (\omega^2 + \alpha^2)^{\frac{1}{2}}\,X \qquad . \qquad 5.01$$

$$\omega_0{}^2 + \ddot{x} = 0 \qquad . \qquad . \qquad 6.02$$

$$\omega_0 = \sqrt{(-\ddot{x}/x)} = \{\text{ acceleration (reversed)} \div \text{displacement }\} \qquad . \qquad 6.04$$

$$\omega_0 = \sqrt{(k/M)} = \sqrt{(\text{controlling constant} \div \text{inertia})} \qquad . \qquad . \qquad 6.07$$

$$T_0 = 2\pi/\omega_0 = 2\pi\sqrt{(M/k)} \qquad . \qquad 6.08$$

$$f = 1/T_0 = \omega_0/2\pi = (k/M)^{\frac{1}{2}} \div 2\pi \qquad 6.09$$

(To be continued.)

❖

Clock-winding in Will. " It is my earnest wish," Mr. Thomas Field, of Aylesbury, founder of the National Association of Goldsmiths, directed in his will " that the firm of Field and Son, Ltd., shall continue the repairs, winding and regulating of clocks at the Royal Bucks Hospital, Aylesbury, and the supply of spectacles to ophthalmic patients in the hospital who are holders of hospital orders, free of charge.

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

(*Continued from page 48*).

9. *Regulating Weights.*

In the usual pattern of clock pendulum, the bob rests on a nut screwed on the end of the rod, and the rate is adjusted by rotating the nut. If the screw has 28 threads per inch, one complete revolution will correspond almost exactly to 40 seconds per day with a seconds pendulum; hence a scale of 40 divisions, combined with a suitable index, enables the rate to be adjusted by an amount which is approximately known. This is a special screw, but OBA, whose pitch is 1 mm. is very nearly right; as near, in fact, as it is possible to make the adjustment.

With fine clocks, such a method is hardly admissible, because the nut cannot be adjusted without stopping the pendulum, which would be intolerable in an observatory. A much better way is to add small weights, either to the top of the bob itself, or on a shelf provided for the purpose about a quarter way down the rod.

Let a mass m be so added at a distance l below the axis of vibration; the moment of inertia is increased by ml^2 and the restoring constant by ml. Hence, the period is altered in the ratio:—

$$T/T_0 = \left[\frac{1 + ml^2/M_T(L_T^2 + r_T^2)}{1 + ml/M_T L_T} \right]^{\frac{1}{2}} \quad (9.01)$$

$$\doteq 1 + \tfrac{1}{2} ml^2/M_T(L_T^2 + r_T^2) - \tfrac{1}{2} ml/M_T L_T \quad . \quad (9.02)$$

$$\doteq 1 - \tfrac{1}{2} ml(L_T - l)/M_T L_T^2 \quad . \quad (9.03)$$

if we neglect r_T, or

$$\Delta \doteq (T_0 - T)/T_0 = +\tfrac{1}{2}(m/M_T)\, l\,(L_T - l)/L_T^2 \quad . \quad (9.04)$$

This increase in the rate is a maximum when the added mass is placed at the mid-point between the axis of vibration and the

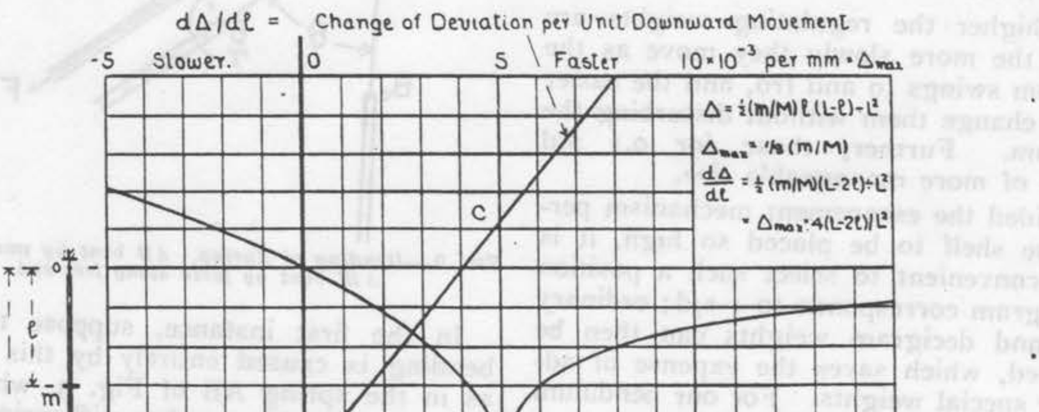


FIG. 8.—*Effect of Regulating Weights on a Seconds Pendulum. A. Relative increase of rate due to adding a given small mass in different positions. B. Relative mass needed in different positions for same increase of rate. C. Relative change of rate caused by moving the mass 1mm. downwards.*

c.g. of the pendulum as a whole; that is when $l=\frac{1}{2}L_T$, and $\frac{1}{2}l\,(L_T-l)/L_T^2=1/8$.

Hence, to get one second per day acceleration in the rate by adding a mass m at the mid-point, we must have:—

$$m/M_T=8\,(T_o-T)/T_o=8/86{,}400 = 1/10{,}800 \qquad (9.05)$$

or approximately one ten thousandth of the mass of the pendulum. A larger mass is of course required when some other position is selected for it.

The pendulum of Table I. has a total mass of 5,070 grams and its c.g. is 978 mm. below the axis of vibration. A mass of 0.47 gram added at the mid-point, 489 mm. below the axis, would therefore produce one second per day increase in the rate. If placed one quarter way down, or 245 mm. below the axis, the mass required would be 4/3 times as great, or 0.63 gram.

The top of the bob is 5in. or 127 mm. above the centre, which is 1,004 mm. from the axis; hence, if the weights be placed on the top of the bob, $l=877$ mm., $(L_T-l)=101$ mm., and the mass needed for 1 s/d is

$$\frac{489\times 489}{877\times 101}\times 0.47 \text{ gram}=1.26 \text{ grams.}$$

The higher the regulating weights are placed, the more slowly they move as the pendulum swings to and fro, and the easier it is to change them without disturbing the pendulum. Further, those for 0.1 s/d become of more manageable size.

Provided the escapement mechanism permits the shelf to be placed so high, it is rather convenient to select such a position that 1 gram corresponds to 1 s/d; ordinary gram and decigram weights can then be employed, which saves the expense of adjusting special weights. For our pendulum this requires $l=133$ mm. or $5\frac{1}{4}$in.

Yet another method is employed in the final rating of an observatory clock. As we shall see, the air slows the pendulum, consequently the rate can be adjusted through a small range by pumping air into or out of the sealed tank in which the pendulum is enclosed. This method is, obviously, not applicable unless the pendulum be hermetically enclosed.

III.—The Suspension Spring.

Geometrically, the most satisfactory support for the pendulum would be a pivot, but the irregularities of its friction make it inadmissible for the accurate measurement of time. In the determination of g by pendulum experiments, a knife-edge is employed, but the fear of trouble due to wear of the edge and to the somewhat indefinite location of the pendulum with respect to the escapement has prevented the application of this method to clocks, in which the pendulum is universally supported by a flexible spring.

The suspension spring gives an equivalent pivot nearly at the mid-point of its free length, but, in addition, it exercises a controlling torque which modifies the rate. The upper end of the spring is held vertical by the clamps, but the lower end must bend round in line with the rod; the spring opposes this bending by the torque just mentioned.

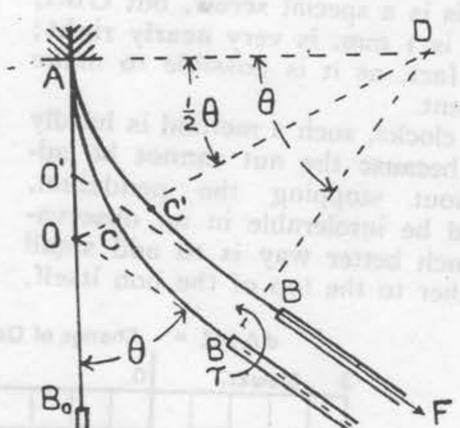


Fig. 9.—Bending of Spring. AB bent by pure torque. AB' bent by force along free end.

In the first instance, suppose that the bending is caused entirely by this torque, as in the spring AB of Fig. 9, which will be nearly true for a rather stiff spring. The bending is then uniform and the spring bends into a circular arc.

Let A and B be the upper and lower ends of the free length of the spring, C its mid-point, and D the centre of the circle into which it is bent. Also, let the centre line of the pendulum cut the vertical through A at O; the symmetry of the diagram shows that O lies on the radius DC produced.

It follows from the diagram that:—

$$AO=\tfrac{1}{2}l\,(\tan\tfrac{1}{2}\theta)\div\tfrac{1}{2}\theta=\tfrac{1}{2}l\,(1+\theta^2/12) \qquad (10.01)$$

With θ as big as 1/20 radian (172 arc-minutes), $\theta^2/12$ is only $1\div 4{,}800$. Consequently, when the spring is uniformly bent, the movement of O as the pendulum swings to and fro is exceedingly small, and we may regard O as a fixed point.

Under this condition of uniform bending, when the pendulum is deflected through an angle θ and the torque is τ, the theory of beams tells us that :—

$$\tau/\theta = bc^3E/12l \quad . \qquad (10.02)$$

where l is the free length of the spring, b its breadth, c its thickness, and E the Young's modulus of the material of which it is made, which may be taken as 30×10^6 lbs. per square inch.

This τ/θ constitutes an addition to the controlling constant, which increases the rate, and whose ratio to the main control is :—

$$(\tau/\theta) \div MgL = (1/12)\,(c^2/l^2)\,(bcE/Mg)\,(l/L) \qquad (10.03)$$

$$= (1/12)\,(c^2/l^2)\,(E/p)\,(l/L) \qquad (10.04)$$

$$= (l/L) \div \lambda^2 \quad . \qquad (10.05)$$

where

$$p = Mg/bc = \text{stress due to weight of pendulum} \quad . \qquad (10.06)$$

$$\lambda = (l/c)\sqrt{(12Mg/bcE)} = (l/c)\sqrt{(12p/E)} \qquad (10.07)$$

The rate deviation due to the spring would be $+(l/L) \div 2\lambda^2$ if the torque were the only thing to be considered.

In addition to the stress p, there is another one due to the bending whose value at the outside layer is :—

$$p_b = \tfrac{1}{2}E\,(c/l)\,\theta \qquad (10.08)$$

The maximum total stress is :—

$$p_t = p + p_b = Mg/bc + \tfrac{1}{2}E\,(c/l)\,\theta \qquad (10.09)$$

Observe that the *thicker* the spring the greater is the danger of its breaking under the bending stresses, other things being constant.

The maximum stress must be kept below the elastic limit of the material, for otherwise each oscillation will add a little more permanent set and the spring will ultimately break.

The elastic limit of good spring steel, after proper heat treatment, is well over 100,000 lbs. per square inch. In selecting a suitable value for the maximum working stress, we must allow for the fact that the actual bending stress is greater than our estimate because the total angle θ is not uniformly distributed along the length of the spring, but is concentrated at the ends for the reasons discussed below. We must also provide a good margin of safety, remembering that during handling for assembly, etc., the spring may be bent considerably more than in actual use.

We may therefore take 35,000 lbs. per square inch as our limit, and set aside 5,000 of that to carry the weight, leaving 30,000 for the bending stress as calculated above.

These considerations set a definite limit to the thickness of the spring for a given length, or to its shortness with a given thickness, when the oscillations are to take place through a given angle. Taking the maximum deflection on each side as 1/20 radian, or about 3°, we get :—

$$\text{Minimum } (l/c) = \tfrac{1}{2}\theta\,E/p_b$$

$$= \tfrac{1}{2}\,(1/20)\,(30 \times 10^6 \div 30 \times 10^3) = 25 \qquad (10.10)$$

In other words, the thickness of the suspension spring of a pendulum should not exceed about 1/25 of its free length, or there may be danger of its breaking in use. In this connection it is important to see that the edges of the clamps are slightly rounded off so that they cannot nick the spring.

In addition to the torque, the pendulum exerts a force on the spring which is approximately equal to the weight and which acts very nearly along the centre line and thus passes nearly through O, as shown in Fig. 10. These actions are balanced by

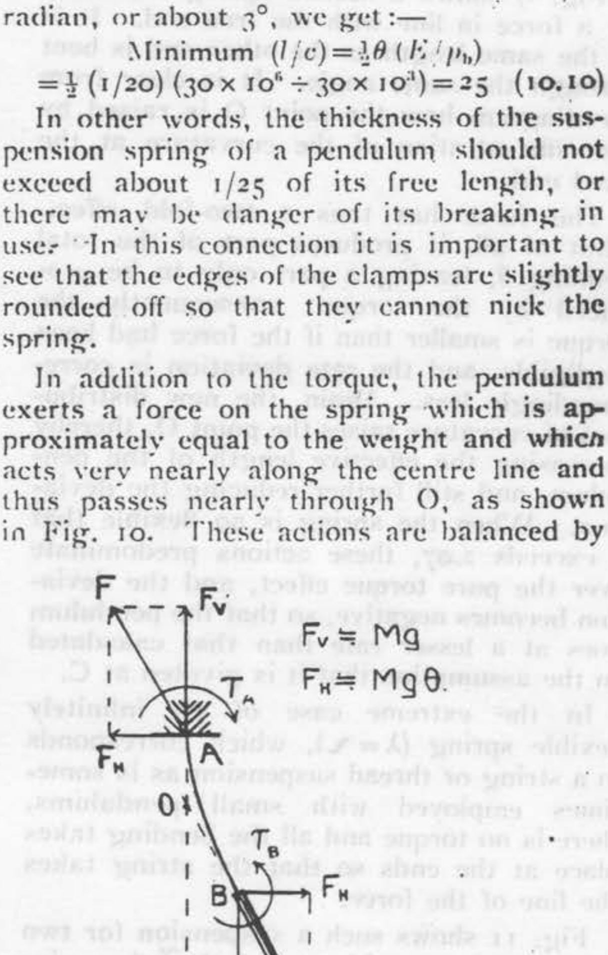


FIG. 10.—*Mechanical Actions on Suspension Springs.*

the opposite reactions at the support, the force being the same (neglecting the weight of the spring) but the torque greater because the first force passes through O and not through A.

Since the force passes below A, it bends the spring in the same direction as the torque with a bending moment at each point proportional to the distance of the spring at that point from the line of action of the force. Thus the bending moment due to the force, and the curvature it produces, are zero at the free end and reach a maximum at the fixed end.

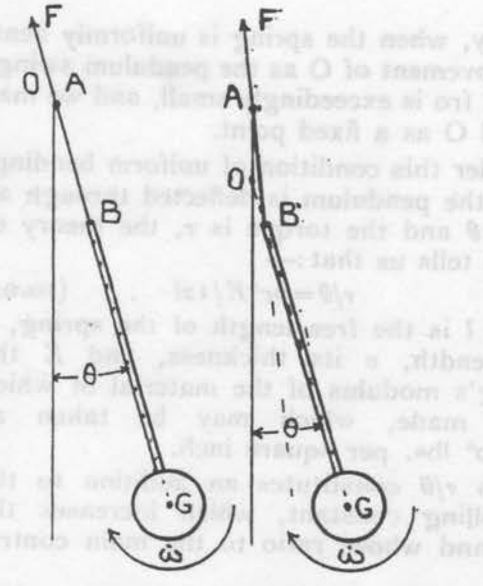
Fig. 9 shows a second spring AB′ bent by a force in line with the free end. It is of the same length as the other and is bent through the same angle. It is clear from the diagram how the point O is raised by the concentration of the curvature at the fixed end.

The force has thus a two-fold effect. First of all, it produces part of the total bending θ, leaving a part only to be produced by the torque; consequently the torque is smaller than if the force had been negligible, and the rate deviation is correspondingly less. Again, the new distribution of curvature raises the point O, thereby increasing the effective length of the pendulum, and still further reducing the deviation. When the spring is so flexible that λ exceeds 2.07, these actions predominate over the pure torque effect, and the deviation becomes negative, so that the pendulum goes at a lesser rate than that calculated on the assumption that it is pivoted at C.

In the extreme case of an infinitely flexible spring ($\lambda = \infty$), which corresponds to a string or thread suspension as is sometimes employed with small pendulums, there is no torque and all the bending takes place at the ends so that the string takes the line of the force.

Fig. 11 shows such a suspension for two pendulums, one without and the other with rotational inertia about an axis through its centre of gravity. In the former case, the string takes the line of the rod and O moves up to A, which we already know to be correct, for this is a "simple pendulum."

Any actual pendulum does have some rotational inertia about an axis through its c.g., and requires a torque to produce the angular acceleration in it. The angular acceleration is always towards the zero position, whether the pendulum is swinging



(a) $\tau = 0$ (b) τ not zero

FIG. 11.—*String Suspension.*

out or in. As the string suspension cannot exert a torque on the end of the rod, it must give the required torque by exerting a force which passes below the c.g., as shown in the diagram. Consequently, the line of the string must be more nearly vertical than the centre line of the pendulum, with the result that O is shifted down from A by an amount which depends upon the radius of gyration of the pendulum about an axis through its c.g.

ERRATA IN PART PUBLISHED IN OCTOBER ISSUE.

The following corrections should be made in the previous article:—

In equations 8.07, 8.08, 8.09, 8.18, 8.19, and also in the corrections to equations 3.03, 6.08 and 6.09, "n" should read "π" (Greek π (pi)).

A multiplication sign should be inserted at the end of the first line of equation 8.18.

In the headings to Table I., "61" should read "G1."

In the correction for p. 13, "Or" should read "OR."

The other "corrections" should be corrected as follows:—

$$\omega_0^2 x + \ddot{x} = 0 \quad . \quad . \quad . \quad 6.02$$

$$\omega_0 = \sqrt{(-\ddot{x}/x)} \quad . \quad . \quad . \quad 6.04$$

(*To be continued.*)

hereby elected members of the Committee *ex officio* :—

Birmingham Jewellers' & Silversmiths' Association, Jewellers' Trade Protection Association, London Wholesale Jewellers' & Allied Trades' Association, Federation of Master Goldsmiths & Jewellers, National Association of Goldsmiths, Edinburgh & East of Scotland Association of Watchmakers, Jewellers and Silversmiths, Scottish Association of Watchmakers and Jewellers, British Horological Institute, Master Silversmiths' Association."

The motion was seconded by Mr. A. F. Klean and carried unanimously.

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

(*Continued from page 73*).

SAMPSON* has given the full theory of the suspension spring, but it is too long to repeat here. He does not put his equations into a form convenient for calculation, and omits to draw any conclusions as to the desirable dimensions for the spring.

His final result can be reduced to the following simple form:—

$\Delta = A\,(l/L)$ (10.11)

where L is the distance between the mid-point of the suspension spring and the c.g. of the pendulum, and A is the following numerical function:—

$A = \frac{1}{2}\lambda^{-1}\coth\lambda - \frac{1}{4}$ (10.12)

Curve

(1). Coefficient A in equation 10.11.

(2). $1/2\lambda^2$.

(3). Continuation of (1) to 10 times smaller scale.

Curve (4). Continuation of (2) to 10 times smaller scale.

The values of this coefficient are plotted in Fig. 12, and also $1/2\lambda^2$ for comparison. It will be seen that the two graphs agree closely with stiff springs, but with very flexible springs the discrepancy is so great that they have opposite signs.

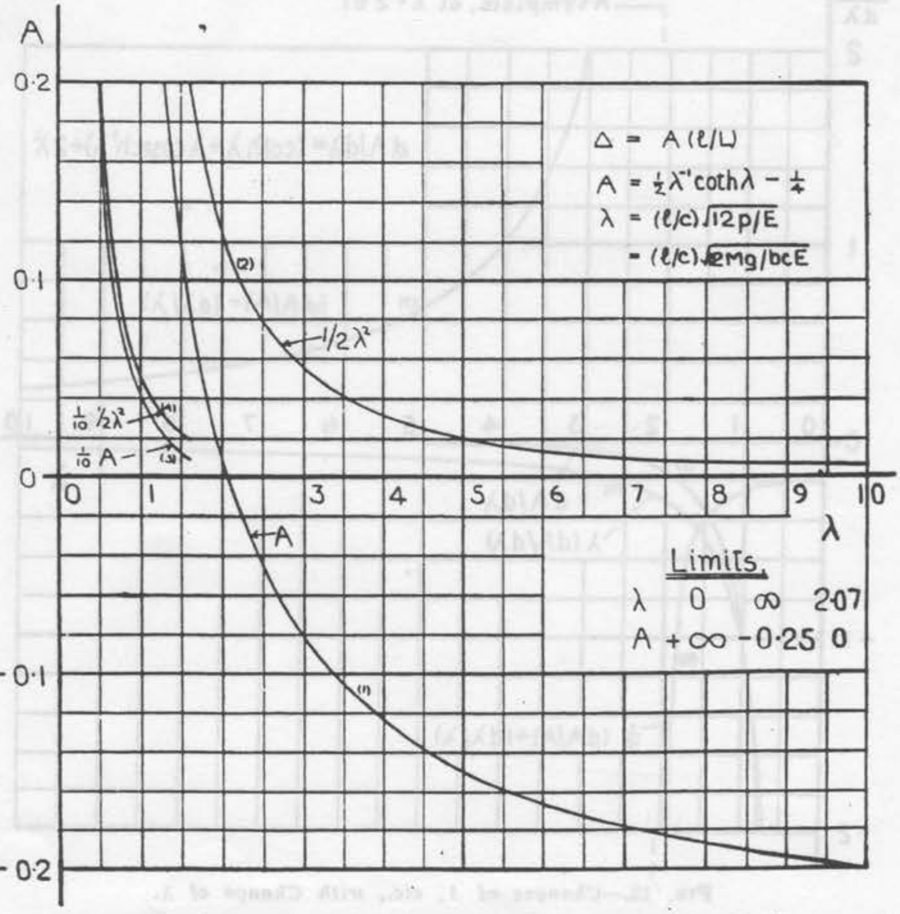


FIG. 12.—*Coefficient for Rate Deviation due to Suspension Spring.*

* R. A. Sampson. "Studies in Clocks and Time-keeping." Proc. Roy. Soc. Edin. Vol. 38, p. 86. January, 1918. If reference be made to this paper, it should be noted that in the numerical examples a decimal point has been misplaced before taking the square root so that his values of λ should be √10 times greater than stated, with consequent large changes in the other numbers derived from them.

11. *Design of Suspension Spring.*

As the correction for the spring is zero when $\lambda = 2.07$, it might be thought that the spring should be designed to have this value as nearly as possible. This is quite wrong. The actual deviation is of no consequence whatever, for it is allowed for in the adjustment of the pendulum by trial. What we require is not zero deviation, but a minimum *change* of that deviation with accidental changes of λ, due to changes in the elasticity, and that is quite a different thing from having no deviation initially.

Curve (5). Slope of curve (1) of Fig. 12 $= dA/d\lambda$.
(6). λ times this, or $\lambda\,(dA/d\lambda)$.
(7). Proportional change of A per unit proportional change of λ or $(dA/A) \div (d\lambda/\lambda)$.
(8). Negative branch of (7) to 10 times smaller scale.

Fig. 13 has been plotted to deal with this point by showing how the rate of change of A, and also λ times that rate of change, vary with the value selected for λ. Both of these functions get smaller as λ is made bigger.

Too long a spring is objectionable, because with it the driving forces cause uncertainties in the position of the pivotting point, which may affect the escapement action. Further, if it be both long and narrow, the pendulum may oscillate too readily about the vertical axis; as such oscillations would be variable, they would interfere with the time keeping.

Consequently, we must not take too great a length simply for the sake of a bigger λ, or we may lose more in one way than we gain in another. Any increase of length affects the ratio (l/L) as well as λ, and when the choice of length alone is under consideration we may take (l/L) as proportional to λ.

Thus, when considering the effect of the choice of length upon the rate error due to a given accidental change of λ, we are concerned with the product $\lambda\,(dA/d\lambda)$, or $dA \div (d\lambda/\lambda)$. But for a given percentage accidental change, which is more likely than a given actual change, we have to deal with the product

$$\lambda dA \div (d\lambda/\lambda).$$

This product has a limiting value of 0.5 when λ is infinite, but nothing appreciable is gained by taking λ greater than 2.5 or 3. The values of the product are 0.595, 0.541 and 0.517 when λ is 2, 2.5 and 3 respectively.

We can thus state definitely that the suspension spring should be as thin as is safe for handling by the type of person who will have to erect the clock, just wide enough to make the frequency of the torsional oscilla t i o n s about the vertical axis about five per second and to keep the stress due to the weight below 5,000 lbs. per square inch, and just long enough to make its λ about 2.5, but

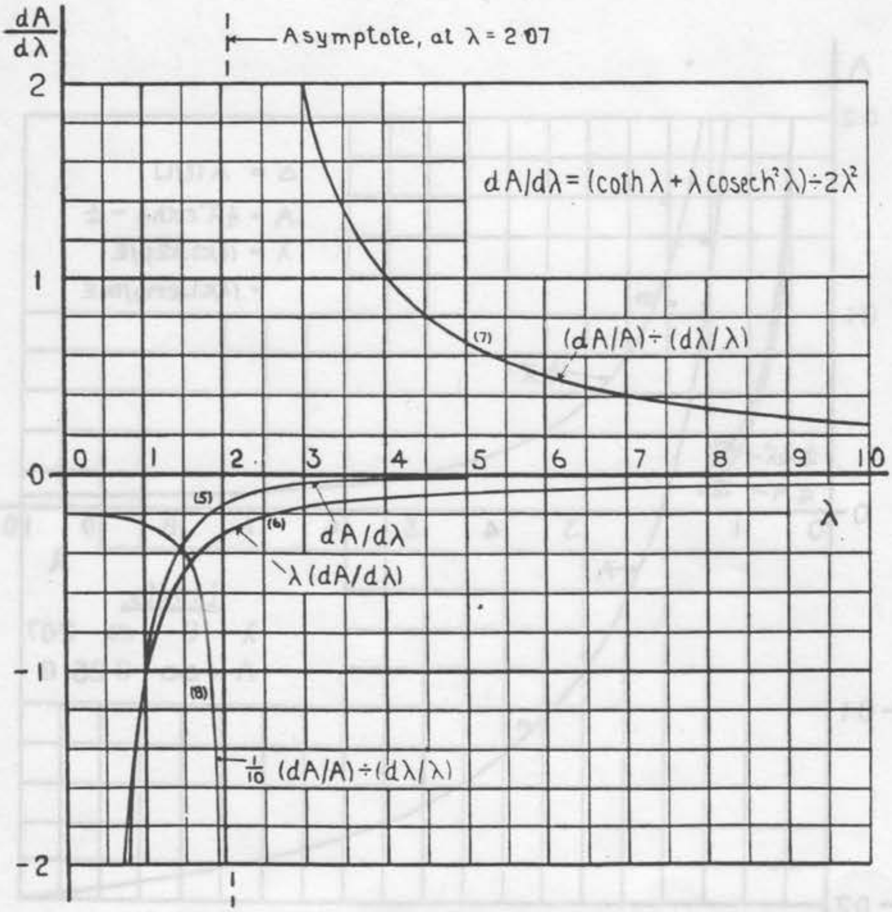


FIG. 13.—*Changes of A, etc., with Change of λ.*

not below 2 nor above 3. This will give a length about twice the minimum for the thickness.

With very thin springs there is a difficulty in fixing them in the clamps without buckling the edges of the hole. For this reason it is desirable to employ material 20 or 30 mils. thick and to grind the free portion down to the desired thickness, leaving the ends of full thickness.

Sometimes trunnions are provided at both ends of the spring, but this is a mistake; the spring should be rigidly attached to the rod and only its upper end supported by a trunnion. With the full leverage of the rod, the friction at the trunnion cannot hold the rod more than about $\frac{1}{4}°$ out of the vertical, but with the much shorter leverage between two trunnions the error at the upper one might easily be as much as 5°, which is more than any reasonable error of clamping with ordinary care.

As we shall see later, the spring accounts for a large part of the temperature error of the pendulum when made of steel. For this reason, the suspension spring of the pendulums of all high grade clocks should be made of the alloy "Elinvar." This is a nickel-chromium steel with about 36 per cent. of the former metal and 12 per cent. of the latter. In addition to having a very low coefficient of expansion, this alloy has an extremely small elasticity-temperature coefficient, and has been developed especially for the springs of balance wheels and pendulums.

Elinvar has a modulus of elasticity of about 27×10^6 lbs. per square inch (18,000 to 19,000 kg. per sq. mm.) and an elastic limit of about 140,000 lbs. per sq. inch (100 kg. per sq. mm.). With it, the maximum working stress should be under 21,000 lbs. per sq. inch (15 kg. per sq. mm.).

12. *Calculations for a Suspension Spring.*

As a numerical example, we shall take the spring used on the pendulum G1 of Table I., which has the following particulars:—

$l = 0.71$ inch $= 18$ mm.
$b = 0.51$ inch $= 13$ mm.
$c = 16$ mils. $= 0.41$ mm.
$M = 5{,}070$ grams $= 11.2$ lbs.
$L = 978$ mm.
$l/c = 18 \div 0.41 = 44.$
$l/L = 18 \div 978 = 0.0184.$
$p = 11.2 \div (0.51 \times 0.016) = 1{,}370$ lbs.-wt. per sq. in.
$p/E = 1{,}370 \div 30 \times 10^6 = 46 \times 10^{-6}.$
$\lambda = 44\sqrt{(12 \times 46 \times 10^{-6})} = 1.03.$
$A = 0.38$ (from graph 1 of Fig. 12).
$\Delta = 0.38 \times 0.0184 = 7.0 \times 10^{-3} = 600$ seconds per day fast.

This deviation is equivalent to a deduction of 14 mm. from the length of the pendulum, and is more than enough to wipe out the 9 mm. which we previously found to be the excess of the effective length of the pendulum over that of a seconds pendulum.

The spring is, in fact, rather a stout one, and has been made deliberately so in order to avoid risk of damage in erection, for the pendulum is for a commercial electric clock which will have to be fixed in place by the ordinary wireman, who is not usually skilled in handling delicate mechanisms.

With an 8-mil spring of the same length and width, λ would be 2.9 and the deviation -122 seconds per day. The spring of pendulum R3 has a λ of 1.62 and a deviation of $+100$ s/d.

It must be understood that the deviation due to the spring cannot be calculated with very great accuracy. The uncertainties as to the exact dimensions of the spring, and especially of its thickness, are too great. Moreover, we cannot depend upon the length of the pendulum being made in exact accordance with the drawings within 10 or 20 mils. One must therefore expect differences of the order of one minute per day between the rate as found on test and that intended, or between that of several pendulums constructed from the same drawings.

IV. Power Required for Maintenance.

13. *Air Resistance.*

During the oscillation, a free pendulum gradually loses its energy of vibration because of the frictional resistance of the air and of any rubbing parts attached to it, and because of the elastic hysteresis of the spring.

The elastic hysteresis loss cannot be separated experimentally and must therefore be lumped with the air resistance; it must be very small, for otherwise the spring would certainly break in the course of time.

Solid friction is approximately independent of the speed, and the power it consumes is proportional to the amplitude if it be in action throughout the whole swing. As such friction appertains rather to the escapement and crutch, if any, than to the pendulum itself, we shall defer the study of its effects on the rate until we come to deal with escapements.

When a body moves through a fluid, there is a resistance to the motion which is zero with zero velocity, but increases as the speed is raised. Below a certain critical speed the motion of the fluid follows smooth stream lines, a condition known as viscous flow, the resistance being termed skin friction.

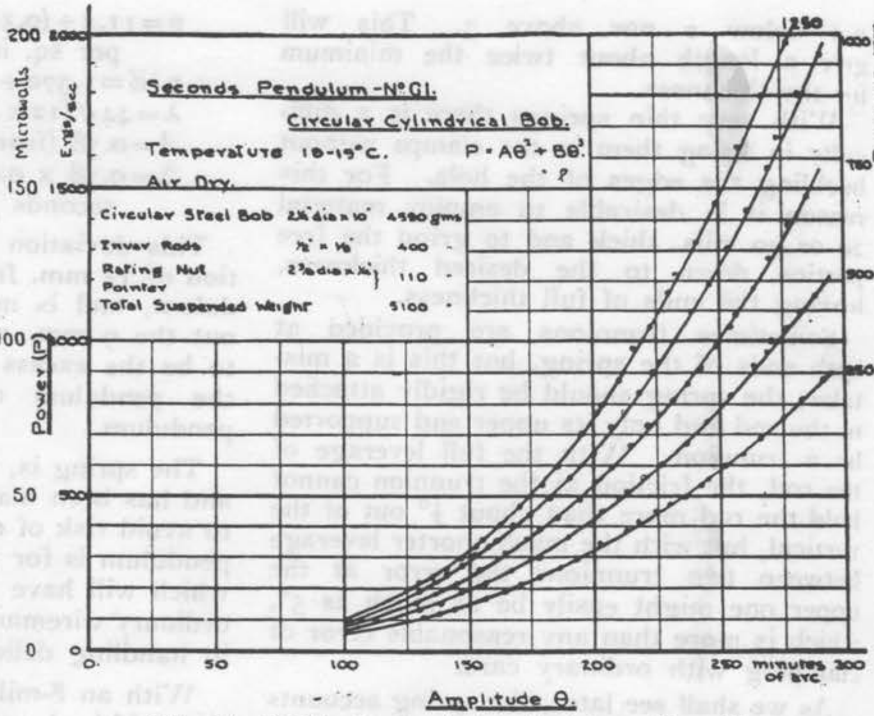


FIG. 14.—*Power consumed by Pendulum G1 with Round Cylindrical Bob. (Hirst.)*

Up to this point the resisting force is proportional to the velocity, to the surface of the body and to the viscosity of the fluid, but does not depend upon the density of the latter. Under these conditions the power consumed is proportional to the square of the velocity.

The critical velocity depends upon the nature of the fluid, and also very greatly upon the shape of the body and of the surrounding enclosure, if any. It is proportional to the ratio of the viscosity of the fluid to its density, but goes down rapidly when projections and rough edges are added to the moving body. Such roughnesses do not affect the resistance below the critical speed, for they are smoothed out by a skin of fluid which moves* with the body, but they not only make eddy resistance begin at a lower speed, but they also raise the amount of that resistance.

Above the critical velocity eddies are formed in the fluid, whose motion now becomes turbulent. We then have eddy resistance which increases much more rapidly than the skin friction and in proportion to some higher power of the velocity. The index varies somewhat with the conditions, but is generally taken as the square, although it may be lower.

Taking the resistance as proportional to the square of the velocity, the power consumed will be proportional to its cube.

In the case of a pendulum, matters are complicated by the fact that during the swing the velocity of each part varies from zero to a maximum, and further, because at each instant the velocity at different distances from the axis of vibration covers the whole range from zero to that at the tip, both maximum values being proportional to the amplitude.

Consequently both skin friction and eddy resistance come into play, except at very small amplitudes when the latter will be absent. It is thus to be expected that the equation for the power needed to maintain the motion of a pendulum will approximate to the type:—

$$P = A\Theta^2 + B\Theta^3 = (A + B\Theta)\,\Theta^2 \qquad (13.01)$$

where Θ is the amplitude and A and B are constants for the given pendulum run under given conditions. Both A and B vary to a large extent with the nature and pressure of the surrounding atmosphere, and they are also affected by the shape of the containing case to a small extent and by that of the bob to a large extent.

Stokes* has worked out the theory of the frictional forces acting on the pendulum

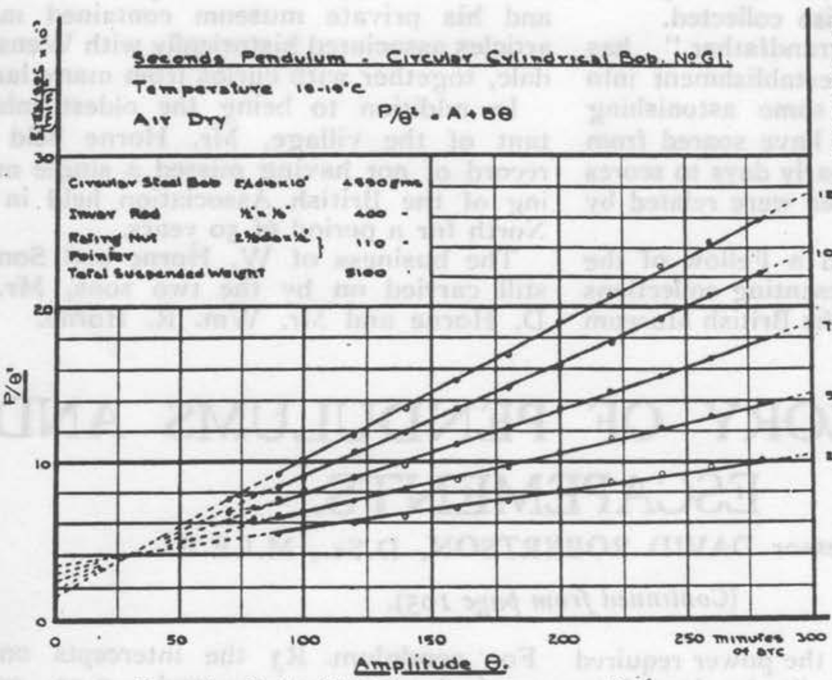


FIG. 15.—*Ratio P/Θ^2 for Pendulum G1 with Round Cylindrical Bob.* (*Hirst.*)

for the two cases of a spherical bob and a rod pendulum, on the assumption that the force is proportional to the velocity, but his results are of little use to us because of that assumption and because they do not apply to the forms of bob that are actually employed. Moreover, his equations are not put into a very convenient form for numerical computation. Nothing but direct experimental data is of real value in this connection.

14. *Power Required with Round Cylindrical Bob.*

A number of determinations of the power consumed by free pendulums have been made by the author's students, more particularly by Mr. A. W. Hirst, M.Sc.,† whose results are given below. These tests were made by the running-down method in which the pendulum is set swinging under the required conditions, and readings of the amplitude are taken at regular intervals during the decay of the motion. The loss of energy between two readings is calculated from the drop of amplitude, and from that the loss per swing is easily determined.

The pendulum was hung inside a jacketed steel tank of $17\frac{1}{4}$in. internal diameter with a clearance of 6in. between the bottom of the bob and that of the tank. A steel scale for the amplitude was fixed immediately behind a fine pointer attached to the bottom of the bob and was observed from some distance through a telescope.

Fig. 14 gives a few of Hirst's graphs for the power consumed by the pendulum of Table I., and Fig. 15 the corresponding values of P/Θ^2. It is evident that the law of equation (13.01) holds very closely indeed for this pendulum down to amplitudes somewhat below 100 arc-minutes. It is noteworthy that the lines for different pressures in Fig. 15 all intersect in one point.

The arrangements did not permit of very accurate measurements further down, but such as were made indicate an approach to a constant value of P/Θ^2 with very small amplitudes. This is what one might expect from the considerations given in section 13, for below a certain amplitude the velocity would never exceed the critical value.

(*To be continued*).

❖

Ambulance Clocks. Public street clocks in Berlin contain in their pedestals an ambulance compartment, where a stretcher and first-aid appliances are always available for emergencies.

* G. G. Stokes. "On the Effect of the Internal Friction of Fluids on the Motion of Pendulums." Camb. Phil. Soc. Trans., ix. p. 8 (paper read 9th December, 1850) and "Mathematical and Physical Papers," Vol. III., pp. 1-141. (Cambridge Press, 1901.)

† Mr. Hirst's experiments were carried out during his tenure of the Proctor Baker Research Scholarship at the Merchant Venturers' Technical College (University of Bristol), the tank and other apparatus being purchased by means of grants to the author from the Colston Research Society, to whom he desires to express his thanks for their aid. The author is also most grateful to Messrs. Gent & Co., Ltd., of Leicester, and to Mr. A. E. Ball of that firm, for the loan of the pendulums G1 and G2, and for assistance in various other ways.

Makers," this long list of names helped to swell the number otherwise collected.

More than one "grandfather" has gone from the Leyburn establishment into Royal households, and some astonishing stories of how the prices have soared from shillings in Mr. Horne's early days to scores of pounds in modern times were related by him.

Mr. Horne was elected a Fellow of the Geological Society for presenting collections of fossils of limestone to the British Museum and the Yorkshire Philosophical Society, and his private museum contained many articles associated historically with Wensleydale, together with curios from many lands.

In addition to being the oldest inhabitant of the village, Mr. Horne held the record of not having missed a single meeting of the British Association held in the North for a period of 50 years.

The business of W. Horne and Sons is still carried on by the two sons, Mr. R. D. Horne and Mr. Wm. R. Horne.

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

(*Continued from page 105*).

IT will be observed that the power required for the ordinary amplitudes is only a few microwatts, or of about the same order as that associated with the reception of wireless signals. Its smallness may be appreciated from the following way of putting it. A penny weighs about 10,000 dynes and about 25,000 ergs of work have to be done in lifting it one inch. To keep the pendulum going with an amplitude of 100 arc-minutes, the penny would have to fall about one foot per hour if the energy of the fall could be imparted to the pendulum without loss. The drop each second would be about equal to the thickness of a rather thin sheet of paper.

As may be seen from the graphs, the power required to maintain a given amplitude increases very considerably with increase of air pressure. Fig. 16 shows how the coefficients *A* and *B* in equation (13.01) vary with the pressure. The increase of *B* agrees tolerably well with the law that the eddy resistance is proportional to the density. The drop of *A* is probably due to the reduction of the critical velocity with increased density, as that causes a smaller portion of the oscillation to be free from eddies.

Tests on another pendulum, No. R3, gave similar results; the *A* and B lines have practically identical slopes, but the former is higher and the latter lower to an extent **sufficient to make *B* negative with low pressures. The negative value of *B* was confirmed by further tests down to 25 mm.**

For pendulum R3 the intercepts on the vertical axes are 5.15 and −9.50, on the scales of Fig. 16, instead of 3.83 and +6.0.

Pendulum R3 has a bob of the same dimensions as that of G1, but there is no rating nut, only a needle pointer below the bob. It has two rods, let into recesses on each side of the bob,* and its suspension

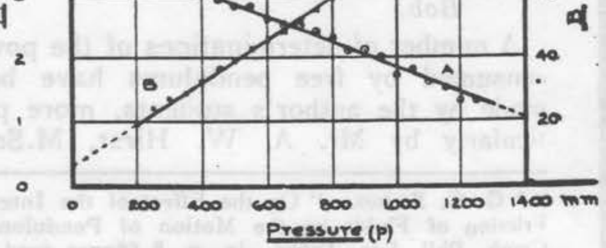


FIG. 16.—*Variation with Air Pressure of Coefficients A and B for Pendulum G1 with Round Bob.* (*Hirst.*)

* The twin rod arrangement enables the driving forces to be applied centrally without a crutch. Drawings of a similar pendulum are given in Plate VII. (p. 278) of the author's paper on "The Clock and Striking Mechanisms of the Great Bell of the University of Bristol," Inst. of Eng. and Shipb. in Scotland. Trans. Vol. LXX., pp. 244-319. Paper read 7th December, 1926.

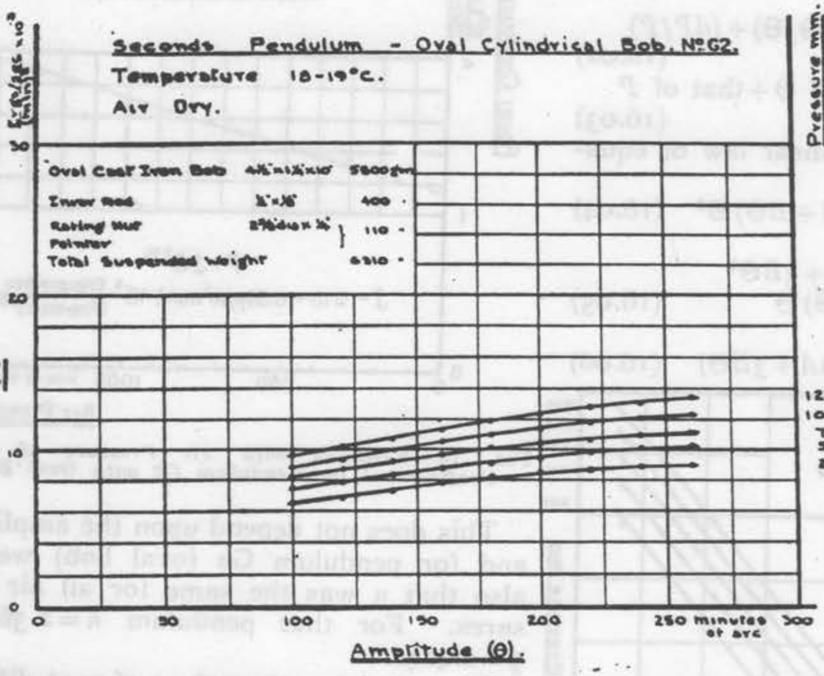


FIG. 17.—*Ratio P/Θ^2 for Pendulum G2 with Oval Bob.* (*Hirst.*)

spring is thinner than that of G1, having the following constants:—$l = 0.5$ in., $b = 0.5$ in., $c = 10$ mils, $\lambda = 1.6$.

Other gases than air have different viscosities and different densities, hydrogen having the lowest value of all gases for both of these properties. The viscosity of hydrogen is about 1/2 and its density about 1/14 of those of air. Consequently the power required to maintain the motion of the pendulum can be very considerably reduced by substituting hydrogen for air in the pendulum tank. No measurements have yet been made with pendulum G1 in any other atmosphere than air, but Schuler† has published graphs showing the decay of the amplitude of his pendulum in air and hydrogen, beginning with an amplitude of 100 arc-minutes. From these graphs it may be inferred that with amplitudes below 100 arc-minutes the resistance of the hydrogen atmosphere is somewhere about 35 to 40 per cent. of that with air. They also show that for his pendulum at these amplitudes the B term is zero, so that the power consumed is proportional to the square of the amplitude. Hence, it is probable that the difference between the two gases would be still greater at large amplitudes where there would be some eddy resistance and the ratio of the densities would come in.

15. *Power Required with Oval Cylindrical Bob.*

Pendulum G2 used the same rod, spring, etc., as G1, but a cast iron bob forming an oval cylinder with rounded top and bottom was substituted for the circular cylindrical bob, the overall length being the same and the masses not very different, 5,800 grams as compared with 4,590. The horizontal section is an oval whose axes are $4\frac{3}{8}$ in. and $1\frac{5}{16}$ in.

Fig. 17 gives the P/Θ^2 curves for this pendulum. This time they are not quite straight and so the law of equation (13.01) does not quite hold, although no serious error would arise from assuming that it did.

Hirst finds that another law,

$$P = J\Theta^{2.36} \qquad (15.01)$$

applies very accurately to this bob, for, as may be seen from Fig. 18, the graphs for $\log P$ and $\log \Theta$ are straight lines.

The slope of these lines is the same for all the pressures tried, showing that the index 2.36 (which is equal to the slope) does not vary with the pressure. The power required does not vary very greatly with the air pressure, which only affects the constant J, as shown in Fig. 19.

It should be noted that the index 2.36 corresponds to a frictional force varying as the 1.36th power of the velocity.

16. *Change of Amplitude with Energy Supplied.*

Under steady conditions, the pendulum must run at such an amplitude that the power dissipated by the friction is equal to that received from the escapement. Consequently any alteration in the amount of power received must result in a change of the running amplitude.

Suppose the power to be increased from P to $(P+dP)$ and that the amplitude is thereby raised from Θ to $(\Theta+d\Theta)$, then:—

$$d\Theta = dP \div (dP/d\Theta) = q\Theta\,(dP/P) \qquad (16.01).$$

† Max Schuler. "Ein neues Pendel mit unveränderlicher Schwingungszeit." Zeits. für Physik., Vol. 42, pp. 547-554. Paper read 8th March, 1927.

where $dP/d\Theta$ is the rate of increase of the power required per unit increase of amplitude, and

$$q = P \div (dP/d\Theta)\,\Theta = (d\Theta/\Theta) \div (dP/P) \qquad (16.02)$$

$$= \text{fractional increase of } \Theta \div \text{that of } P \qquad (16.03)$$

Take the case of the linear law of equation (13.01), namely:—

$$P = A\Theta^2 + B\Theta^3 = (A + B\Theta)\,\Theta^2 \qquad (16.04)$$

Then

$$dP/d\Theta = 2A\Theta + 3B\Theta^2$$
$$= (2A + 3B\Theta)\,\Theta \qquad (16.05)$$

and

$$q = (A + B\Theta) \div (2A + 3B\Theta) \qquad (16.06)$$

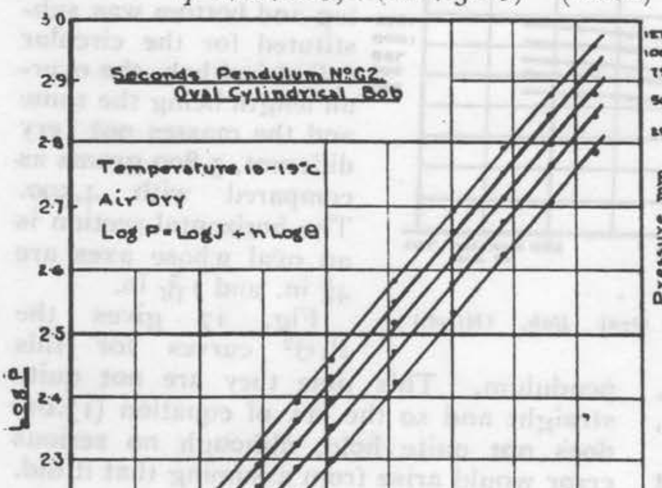


FIG. 18.—*Log P and Log Θ for Pendulum G2 with Oval Bob. (Hirst.)*

The number q lies between the limits 1/2 when $B = 0$, or there is no eddy resistance, and 1/3 when $A = 0$, or the skin friction is negligible. In other words, a change of 1 per cent. in the power supplied will cause anything between 1/2 and 1/3 per cent. change in the amplitude according to the characteristic of the pendulum at the point where it is working.

For the exponential law of equation (15.01), namely:—

$$P = J\Theta^n \qquad (16.07)$$

$$dP/d\Theta = nJ\Theta^{n-1} = nP/\Theta \qquad (16.08)$$

$$q = 1/n \qquad (16.09)$$

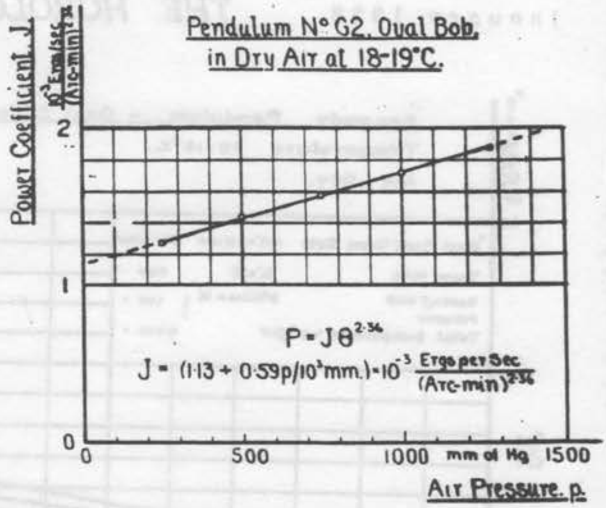


FIG. 19.—*Variation with Air Pressure of Power Coefficient J for Pendulum G2 with Oval Bob.*

This does not depend upon the amplitude, and for pendulum G2 (oval bob) we saw also that n was the same for all air pressures. For that pendulum $n = 2.36$ and $q = 0.424$.

Fig. 20 shows the values of q at different amplitudes for pendulum G1 at two air pressures, and the constant value for G2 applicable to all pressures.

17. *Change of Amplitude with Change of Air Pressure.*

When running in an unsealed case a pendulum loses amplitude when the barometer rises, the change being such that the reduction of the power needed which is brought about by the fall of amplitude balances the increase due to the rise of pressure, or vice-versa. Thus, when the pressure rises from p to $(p + dp)$, we have:

$$d\Theta\,(dP/d\Theta) = -dp\,(dP/dp) \qquad (17.01)$$

where dP/dp is the increase, per unit in-

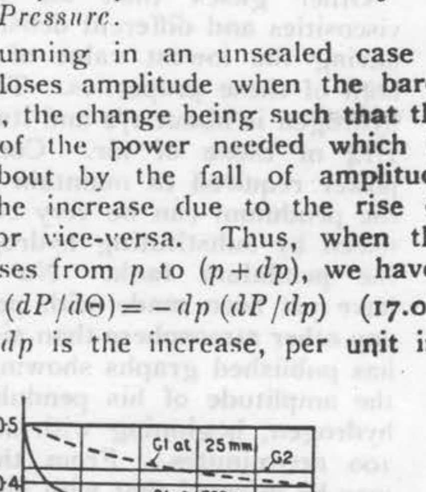


FIG. 20.—*Ratio of Fractional Changes of Amplitude and Power for Pendulums G1 and G2.*

crease of pressure, of the power required to maintain a given amplitude. Hence,

$$d\Theta = -dp \{ (dP/dp) \div (dP/d\Theta) \} \qquad (17.02)$$

Using the linear law of equation (16.04), we get:—

$$dP/dp = (dA/dp + \Theta dB/dp)\, \Theta^2 \qquad (17.03)$$

Now, dA/dp and dB/dp are the slopes of the lines for A and B given in Fig. 16; they are equal to the coefficients of p in the equations to those lines which are given at the foot of the diagram. Thus, for pendulum G1,

$$dA/dp = -2.0 \times 10^{-6} \{ \text{(ergs/sec.) per arc-min.}^2 \} \text{ per mm.} \qquad (17.04)$$

$$dB/dp = 0.067 \times 10^{-6} \{ \text{(ergs/sec.) per arc-min.}^3 \} \text{ per mm.} \qquad (17.05)$$

Fig. 21 shows the values of $d\Theta/dp$ calculated from these figures.

For the law of equation (16.07), which applies to the oval bob and in which the index n is independent of the pressure,

$$dP/dp = (dJ/dp)\, \Theta^n \qquad (17.06)$$

$$d\Theta = -dp \{ (dJ/dp)\, \Theta^n \div nJ\Theta^{n-1} \} \qquad (17.07)$$

$$= -dp \{ (dJ/dp)/nJ \}\, \Theta \qquad (17.08)$$

From the equation at the foot of Fig. 19 we get:—

$$(dJ/dp)/nJ = 0.59 \times 10^{-3} \text{ per mm.} \div 2.36\,(1.13 - 0.59\, p/10^3 \text{ mm.})$$

$$= 0.59 \times 10^{-3} \text{ per mm.} \div 2.36 \times 1.57 \text{ when } p = 750 \text{ mm.}$$

$$= 0.159 \times 10^{-3} \text{ per mm.} = 4.03 \times 10^{-3} \text{ per inch.}$$

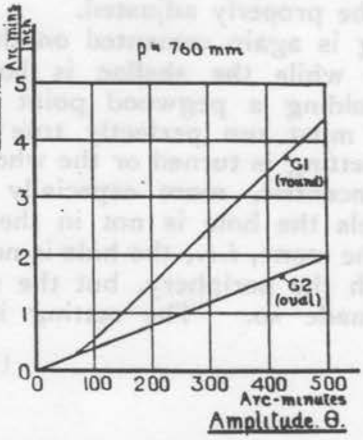


FIG. 21.—*Change of Amplitude with Change of Barometric Pressure for Pendulums G1 and G2.*

The graph for this pendulum is also shown in Fig. 21. The difference between the two pendulums is somewhat less than might be expected from the relative magnitudes of the power required to maintain a given amplitude because of the smaller q for the oval bob.

(To be continued.)

The Lady Watch Repairer.

THE enterprising girl who wishes to start out upon a new career will find plenty of scope, provided she has dainty hands, as a watch and clock repairer.

The work is suited to a girl who wishes to have a career in which she is quite "on her own." She could run her enterprise in her own home, and probably combine a certain amount of visiting work, especially if she included ordinary jewellery repairs in her training.

The student would have to go as an assistant to a firm for two or three years to learn the trade. During that period she would receive quite a small salary, and for that reason the younger she is at the start the better. There is theory to be learnt as well as the practical work, and the future watch and clock repairer would find it an advantage to attend a course of evening classes in her subjects at one of the polytechnics, several of which have such classes.

Her aim should be to pass the examinations of the Horological Institute. The fees are quite low, the highest being only one guinea. When she was thus qualified she could assure clients that she was perfectly able to undertake their work. Nowadays certificates count for much.

Once the training was over and the examinations were passed, she would have to consider her next step.

It would be a good plan to go to a wholesale house for six months in order to gain a further insight into the trade. After that she could start on her own with very little capital, just sufficient to buy the necessary apparatus and to enable her to advertise.

While building up her connection she would find it helpful to work for some of the big shops. During the time she was an assistant she would have learnt the "ropes" of the trade, and so would know how to set about getting this work.

Daily Telegraph.

fit into these spaces. It is best to do this in pairs, starting with the 'scape arbor first, fitting first the back and then the front jewel; by so doing the end shake of each arbor can be properly adjusted.

The plug is again cemented on the face plate, and while the shellac is hot, run true by holding a pegwood point in the hole; this must run perfectly true before the brass setting is turned or the whole will not be concentric, more especially as in heavy jewels the hole is not in the exact centre of the stone, *i.e.*, the hole is not concentric with the periphery, but the setting must be made so. The setting is now turned down till it fits the small hole in the plate as far as the ledge, *i.e.*, 3/32in., and the large end is then turned down till it fits the large hole in the plate. It is then removed from the face plate and reversed, the end in which the jewel is set being cemented to the plate, run true, and the face turned down sufficiently for it to stand above the plate 1/32in., so that the heads of the screws which are to hold it in place may press against it. The hole in the brass may be turned out cone-shaped to within 1/16in. of the edge, leaving a rim against which the screw heads press.

(To be conluded.)

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

(Continued from page 131).

18. *Effect of Temperature on the Power Absorbed.*

No power-temperature tests were carried out with pendulums G1 or G2, but Fig. 21 gives the results of four such tests made by Mr. Hirst on pendulum R3, and Fig. 23 gives the corresponding variation with temperature of the coefficients *A* and *B*. Dotted lines have also been drawn on the latter figure to show how the viscosity and density of air vary with the temperature, to such a scale as to make these lines lie very close to the others at 0°C.

It will be observed that both the *A* and *B* curves lie very nearly parallel to the density line; that is to say, that both these coefficients vary with the temperature in almost the same ratio as the density, whose temperature coefficient is -3.665×10^{-3} per °C. So far as *B* is concerned, that is exactly what is to be expected from the general facts about turbulent motion given in section 13, but it is rather surprising that the *A* line should follow the same law instead of rising with the viscosity.

The *q* for this pendulum is not very different from that of G1 as given in Fig. 20. Hence we may conclude that for pendulum R3, supplied with constant power, the temperature coefficient of the amplitude will lie between 1.0 and 1.5×10^{-3} per °C., the amplitude rising when the temperature goes up.

Until much more experimental evidence is available it must not, however, be supposed that these figures apply, even approximately, to pendulums in general.

A few tests were made on the effect of

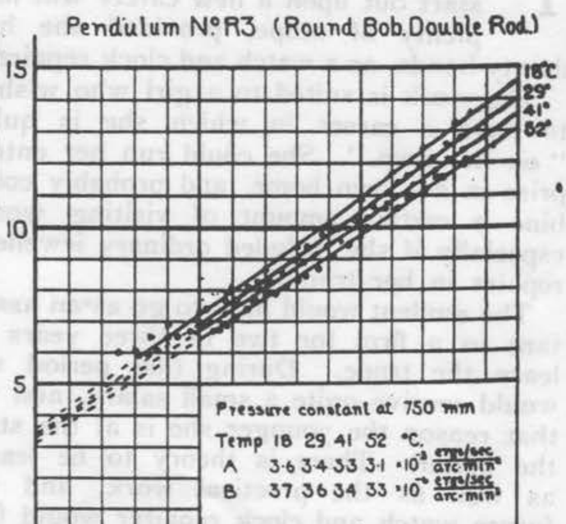


FIG. 22.—*Power required at Different Temperatures by Pendulum R3. (Round Bob, Twin Rods.)*

same accumulator also operates a Morse recorder, used to receive automatically the six-dot time signal—along with clock time—for comparison.

As the microphonic relay is from its nature susceptible to mechanical vibration, it should be mounted on rubber feet or on springs, and on no account must the electromagnetic relay be mounted on the same base, because the movement of its armature would—under conditions of adjustment—operate the microphonic relay and set up continuous chattering, due to interaction.

The rod R may be of non-oxidable metal instead of hard plumbago. Such arrangement, however, makes the relay rather more susceptible to mechanical vibration, but slightly increases its electrical efficiency.

(To be continued.)

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

(Continued from page 163).

IV. Barometric Errors.

21. *Atmospheric Deviation.*

THE atmosphere surrounding the pendulum affects its rate in three ways, namely:—

(*a*) A reduction of effective weight by the bouyancy of the air; this slows the pendulum.

(*b*) An increase in the effective inertia owing to some air moving to and fro with the pendulum; this also reduces the rate.

(*c*) Air friction reduces the rate, but to an entirely negligible extent.

The buoyancy is equal to the weight of the air displaced and is therefore quite easily calculated. At a temperature of 15°C. (59°F.) and a barometric pressure of 700 mm., the density of air is 1.225 grams per litre. As that of type metal is about 10 kilograms per litre, 1/8200 of the weight of a bob of this metal is wiped out by the buoyancy of the air; the controlling constant is thus reduced to that extent by the buoyancy, and the rate by half as much, or 1/16 400, which is 5.3 seconds per day.

For bobs of other materials the deviation of the rate by the air buoyancy is inversely proportional to their density. The barometric error is consequently reduced by employing a bob of high density, which has the further advantage of increasing the ratio of the energy of the vibration to that required to maintain it. A few examples are given in Table II.

TABLE II.

Buoyancy Deviation at Atmospheric Pressure.

Metal.	Steel.	Brass.	Type-Metal.	Platinum
Specific Gravity	7.8	8.5	10.0	21.5
Buoyancy deviation at 760 mm. ... 10^{-6}	−70	−72	−61	−28
Buoyancy deviation at 760 mm. ... s/d...	−0.8	−0.2	−5.3	−2.5
Buoyancy error at 760 mm. 10^{-3} s/d. per min.	−9.0	−8.2	−7.0	−3.3
Buoyancy error at 760 mm. s/d. per inch ...	−0.23	−0.21	−0.18	−0.084

The air friction not only opposes the motion of the pendulum, but it drags some air with it, and in this way adds to the inertia. The exact amount of this extra inertia cannot be calculated with any accuracy, because it varies with the form of the pendulum, with the size and shape of the enclosure, and possibly also with the amplitude. The only certain method is by direct experiment.

In the paper already mentioned (page 105) Stokes has worked out its value in the cases of spherical pendulums and plain round rod pendulums, on the assumption that the frictional force is proportional to the velocity. His results give values of the ratio (inertia effect ÷ buoyancy effect) of 0.5 for large spheres to 1.2 for one of 20 mm. diameter, and of 1.0 for cylinders of very large diameter to 1.24 for one of 40 mm. In both cases the ratio becomes very large with very small diameters.

Relying on Stokes' theory, it is commonly assumed that the inertia deviation is about equal to the buoyancy deviation, or the joint effect about twice the latter.

Balance wheels and torsional pendulums are not controlled by gravity but by the elasticity of the spring or wire. Consequently they are not affected by the buoyancy, but the air does add to their effective inertia, and consequently they are not quite free from barometric errors. With a torsional pendulum the amount must be extremely small, but with a balance wheel the volume of air affected will probably be considerably greater than that displaced by the rim, and more nearly equal to that displaced by a solid wheel of the same diameter and thickness.

22. Barometric Errors.

The atmospheric deviation is allowed for unconsciously when the pendulum is adjusted to the correct rate by trial in the usual manner. Its amount is therefore of no consequence whatever so long as it is kept constant. With observatory clocks, this condition is secured by placing the whole mechanism inside a hermetically sealed tank, and no other arrangement would ever be permitted where accurate timekeeping is the primary consideration.

But a sealed clock is costly, and it is not applicable to ordinary purposes for various other reasons. Consequently, the great majority of clock pendulums have to run in air whose density varies wtih each rise and fall of the barometer, and thus they have a barometric error.

A rise of 1in. in the barometer would cause the atmospheric deviation to increase by about 1/30 of its amount. With the type metal bob, the buoyancy error is thus −0.18 s/d. per inch. The corresponding figures for the other bobs are given in Table II. The combined buoyancy and inertia error will usually lie between 1.5 and 2.5 times the value for the buoyancy alone.

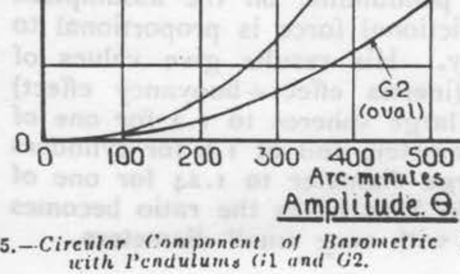


FIG. 25.—*Circular Component of Barometric Error with Pendulums G1 and G2.*

But a rise of air density does more than alter the atmospheric deviation; it reduces the amplitude and thereby causes a circular error, and also timing and other indirect errors by modifying the deviations due to the escapement forces.

The drop in amplitude reduces the negative circular deviation, and therefore gives a positive barometric circular error, which is opposite in sign to the buoyancy and inertia errors. As its amount is $(1/8)\,\Theta d\Theta$, the circular error is easily calculated when the data required for getting Fig. 21 are known. Fig. 25 shows the circular component of the barometric error of the two pendulums G1 and G2 calculated from the values of $d\Theta/dp$ given in Fig. 21.

Since the circular error rapidly increases with increase of amplitude, whereas the other two are nearly, if not quite, independent of amplitude, it is obvious that there is one amplitude at which the resultant of the three is zero. If we assume that the buoyancy and inertia errors together amount to 0.4 s/d., this amplitude would be 240 arc-minutes for G1 and 330 for G2.

There still remain the amplitude and timing errors due to the changes in the escapement deviations. No general statement can be made concerning these errors, for both their amount and sign depend upon the arrangement of the escapement and upon its adjustment. They may, or may not, be of the same sign as the buoyancy error, or they may even be of negligible amount. They must be worked out separately for each case.

Tests were made to find the effect on the rate of pendulum R3, of varying the air pressure with the pendulum running steadily on its own escapement with a constant supply of power. The amplitude at each pressure was also noted and the results are shown in Fig. 26, to which has been added the escapement and circular errors corresponding to the change of amplitude.

The amplitude at each pressure was not very steady, owing to variations of escapement friction, but the greatest observed at

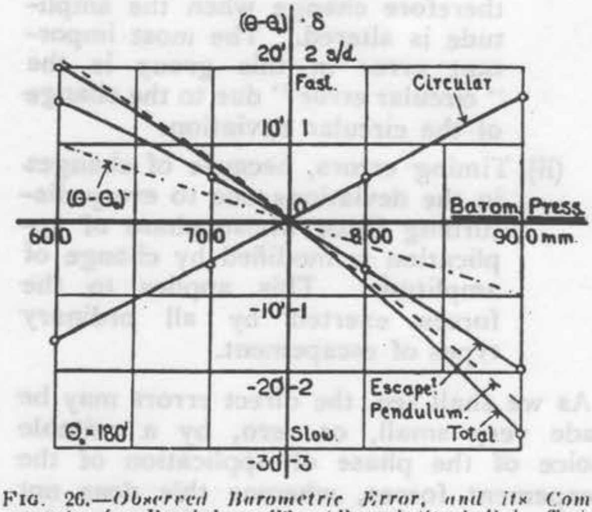


FIG. 26.—*Observed Barometric Error, and its Components, for Pendulum R3. (Round Steel Bob, Twin Rods.)*

each pressure was taken as the representative one. The variations of amplitude agree with those calculated by the method used for Fig. 21 much more closely than the conditions of the test would lead us to expect.

It will be noted that with the particular setting of the escapement employed during the test, the circular and escapement errors very nearly balance one another, and the observed error is but little different from that due to the pendulum alone. The graphs for the errors are shown curved, but the accuracy of the test is not sufficient to justify any statement about this curvature other than that it may be caused by errors of experiment.

The mean values for the error coefficients given by this test are:—

	ms/d per mm.		s/d per inch.
Total error	−16	or	−0.41
Due to pendulum alone..	−15	,,	−0.38
Buoyancy, from Table II.	−9	,,	−0.23
Inertia	−6	,,	−0.15

Many attempts have been made to get zero total barometric error by balancing the other components by the circular error. They had not much chance of success, for the estimate of the correct amplitude was generally based upon incomplete or erroneous data, and all the factors were not always taken into account. We now see how to make the calculation, but we also find that the amplitude necessary is so large that the circular errors due to other variations, such as a change of the pallet friction, may produce much larger errors than the barometric ones which have been eliminated.

With the same total pressure, the density of the air is lower the greater its absolute humidity. The maximum range of humidity in this country would give about the same effect as a ¼in. change of barometer. Due to this cause, there may therefore be a change of rate of about 0.1 s/d. between dry frosty weather in March and a humid day in August, the rate being faster with the humid atmosphere.

(*To be continued*).

Practical Column.

By J. W. PLAYER.

"These little incidents will occur even in the best regulated workshops."

This column is published with the idea of giving practical assistance to watch repairers and others who seek a solution of any difficulty they have to contend with, whilst at the same time the answers to questions will be found of great interest and help to other readers. All questions should be addressed to The Editor, Practical Column, Horological Journal, Fulwood House, High Holborn, London, W.C.1, by the 7th of the month for reply in the following month's Journal.

"LINCOLN" asks:—"May I request that you will supplement your advice of a few months ago *re* selecting a hairspring for a watch with some instruction on trueing the hairspring in the eye; that is a point which all books mention, but none that I have seen give any practical help in the matter. What I want to know is this:—How can one make absolutely certain that the flat spring is accurately centred at the pivot?"

Answer.—This is one of those cases where one finds prevention much better than cure, for if the spring is cut out correctly for the collet and is properly put on at the start, the business of trueing it at the eye is very simple.

The spring should not hug the collet; it should come out quite clear of it; on the other hand, if too much spring is taken out, it becomes necessary to bend it inwards just enough to make up for this, and to do so is not always as easy as it looks, and may lead to a certain amount of experimental bending backwards and forwards, which is bad for the spring in any case, as it disturbs its elasticity at a vital part; besides, there is always the risk of giving

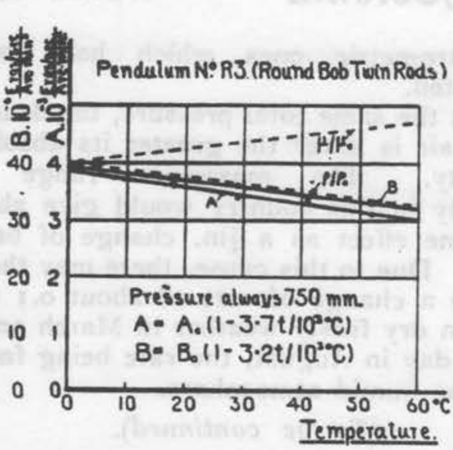


FIG. 23.—*Variation with Temperature of Coefficients A and B for Pendulum R3. (Round Bob, Twin Rods.)*

humidity on the power required by pendulums G1 and G2, but the results were not very conclusive, and it was suspected that they were vitiated by some time effect, such as condensation on the bob and subsequent re-evaporation. With the air saturated with moisture at about 18°C. and the same total pressure, the round bob took from 0 to 5 per cent. more power than with dry air; with the oval bob the differences were smaller and sometimes of the opposite sign.

19. *Relation between Power Characteristics and Errors.*

In order to emphasise the importance of the power characteristics of the pendulum, we shall anticipate the results of the late chapters by giving a brief statement concerning the escapement errors.

Any accidental variation in the amount of an escapement force, or any other disturbing one, or in the phase of its application, will cause a whole series of erors in the rate of the pendulum. These errors may be grouped as follows :—

(*a*) Direct error, because the quantity which has changed is a factor in the deviation caused by the force affected.

(*b*) Indirect error, because the original accidental change alters the energy supplied and hence the running amplitude. The indirect errors may be sub-divided into two sub-groups :—

(i) Amplitude errors, because the deviation due to each disturbing force, including the one concerned with the original variation, is proportional to the amplitude and must therefore change when the amplitude is altered. The most important error in this group is the "circular error" due to the change of the circular deviation.

(ii) Timing errors, because of changes in the deviations due to every disturbing force whose phase of application is modified by change of amplitude. This applies to the forces exerted by all ordinary types of escapement.

As we shall see, the direct errors may be made very small, or zero, by a suitable choice of the phase of application of the escapement forces, whereas this does not apply to the indirect ones. Consequently, the indirect errors, and particularly the circular error, caused by some accidental change, may be considerably greater than the direct error due to the same change.

Further, the amount of the direct error for any assumed variation is proportional to the ratio (energy required per swing) ÷ (energy of vibration), which is itself proportional to P/Θ^2. The indirect errors take no account of this ratio but are proportional to the one we have already called q, namely, (fractional change of amplitude) ÷ (fractional change of power supplied).

For the smallest possible direct errors, we require a minimum value of P/Θ^2, but for least indirect errors we want q to be as small as possible. Unfortunately, when we reduce the former by a change in the shape of bob or in the conditions of running, we generally increase the value of q.

In the absence of lengthy detailed calculations for the escapement to be employed, and a reliable estimate as to the particular kind of accidental change which is likely to interfere most seriously with the time-keeping, it is impossible to say, except in extreme cases, whether the gain on the one count is greater or smaller than the loss on the other.

For example, by reducing the air pressure P/Θ^2 is made smaller, but q is made greater. Beginning with atmospheric pressure, the total error is reduced by choosing a lower air pressure, but beyond a certain point, which varies with the characteristics of both the pendulum and the escapement, it is increased by going to still lower pressures.

The circular error is proportional to $q\Theta^2$. Consequently the amplitude should be as small as other circumstances permit. But the timing error, due to wear, obliquity combined with end shake, etc., of the pallets, makes it desirable to have a large amplitude, and so some compromise is necessary. This last mentioned error may be assumed to be the one which limits the ultimate accuracy after allowance is made for the secular change in the length of the rod, which is rather appreciable in the case of invär.

20. *Comparison of Round and Oval Bobs.*

Fig. 24 shows how the power taken at several amplitudes varies with the pressure

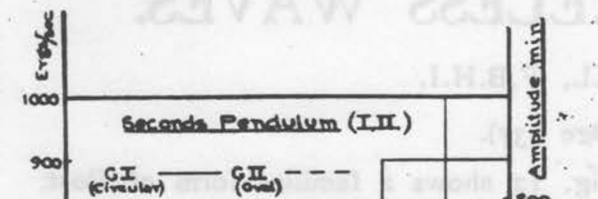


FIG. 24.—*Comparison of Power taken by Round and Oval Bobs. (Hirst.)*

for both the round and oval bobs. The two lines for one amplitude cross in points whose locus is shown by the dotted line. With a given amplitude, the round bob requires more power than the oval one when the pressure is above that at the crossing point, but with lower pressures the oval bob takes more.

Similar results can be obtained by superimposing Figs. 15 and 17. With a given air pressure, the two bobs take the same power at one particular amplitude. At higher amplitudes, the circular bob takes the greater power, but with lower amplitudes, the oval bob. At atmospheric pressure, the amplitude at the crossing point is about 100 arc-minutes.

The oval bob has a greater surface than the round one, but it more nearly approaches the streamline form. Consequently it has the bigger skin friction, but a smaller eddy resistance. With high air pressures, or large amplitudes, where the eddy resistance forms the bulk of the total, the reduction of that portion by employing the oval bob exceeds the increase in the skin friction and the total resistance is smaller. But with low pressures, or small amplitudes, where the eddy resistance is much less in proportion, the reverse is true.

Thus, the oval bob has no advantage even with respect to the direct errors, whilst with respect to the indirect errors its greater q is a serious disadvantage as compared with the round bob.

For unsealed pendulums, the smaller value of $d\Theta/dp$ shown in Fig. 21 is also undesirable, because the circular error due to the change of amplitude when the barometer falls or rises, is of the opposite sign to the other errors due to that cause, and less than these other errors; it thus helps to wipe them out, and so to reduce the total barometric error.

There are other objections to the oval bob. When mounted, its major axis is not likely to lie truly in the plane of the vibration, and that position is unstable with respect to the air frictional forces. The air resistance causes the pressure at the entering edge to exceed that at the leaving edge, and consequently tends to turn the bob flatwise to the motion instead of edgewise. This gives a possible cause of oscillations about the vertical axis, which would interfere with the regularity of the rate.

We may therefore conclude that the oval bob should never be employed, but that the shape of the bob should always be a surface of revolution about the vertical axis.

Certain authorities recommend that the bob should be a circular cylinder whose length is equal to its diameter, and it seems probable that this shape is very nearly the best possible. For a given volume, it has the least surface of all proportions of cylinder, and has therefore the lowest skin friction. It is just a little worse in this respect than the sphere.

It is by no means certain that it is improved by rounding off the edges; by doing so, the total resistance would be reduced, but the q would be raised. For the same reason, it is doubtful whether the sphere is better than a cylinder of the proportions given in spite of its somewhat smaller surface.

Fig. 24 brings out another point by showing the futility of seeking to obtain the spring loss by producing the lines to zero air pressure. These two pendulums had the same individual spring and so would have identical spring losses with the same amplitude. But the intercepts on the axis of zero pressure differ in the ratio of about 3 : 2, instead of being alike. The fact is that the air friction does not diminish in such a way as to become zero at zero pressure, because the viscosity remains constant until exceedingly low pressures are reached, pressures of the order of a fraction of a millimetre of mercury.

(*To be continued.*)

THE AUTOMATIC SYNCHRONISATION OF CLOCKS BY WIRELESS WAVES.

By ALFRED E. BALL, F.B.H.I.

(*Continued from page* 137).

CLASS "B" SIGNALS—*continued*.

Sensitive Relays. Moving Iron Type.—As the name implies moving iron relays are fitted with iron armatures which are moved by the operating current in contradistinction to a coil of wire being so moved as in the case of the moving coil type of relay last described. In a moving iron relay the moving armature itself has no coil or windings, such windings being on the poles of a stationary electro-magnet, and consisting of very many turns of fine wire, consequently resulting in a high resistance, usually from 3,000 to 5,000 ohms or higher. Contrasted with the moving coil relay, such a high resistance is not required, 300 or 400 ohms being usual.

In order that a moving iron relay may come under the title "Sensitive," it must have its armature and its magnet poles polarised by having adjacent to it a strong permanent magnet. This polarised construction has the effect of making the relay about four times as sensitive as a non-polarised relay of same size and with same windings.

The reason of the increased sensitiveness lies in the fact that the "weakened by air gap" effect of the non-polarised relay is absent, because the armature is already in a strong field, which is by the incoming signals weakened at one pole and simultaneously strengthened at the other.

Fig. 12 shows a familiar form of Post Office polarised relay and Fig. 13 shows diagrammatically its construction.

This type of relay is usually mounted in a cylindrical brass case and the upper part of the case is hinged to give access to that part of the relay which requires setting and adjustment. The top of the case is glazed to make its action visible when closed. The polarising permanent magnet is curved to conform to the shape of the case. The armature can also be "biased" magnetically on one side or the other by turning the milled nut seen on the right of Fig. 12.

These relays are particularly well made and may be purchased fairly cheaply from disposal firms; but before they can be used with advantage for operating by wireless, the existing windings of comparatively coarse wire must be removed and the ebonite bobbins rewound with fine wire, such as No. 43's, so as to obtain a large number of turns. The resistance should be about 5,000 ohms.

Fig. 13 shows diagrammatically its construction. B,B1 represents two poles of the electro-magnet and S S two pole pieces of same. B2, B3, show two ebonite bobbins which have to be wound with many turns of fine wire, the resulting resistance being about 5,000 ohms. C shows a contact arm which is mounted on an arbor and pivotted at C1, and on the same arbor is mounted a soft iron armature N.S., which is, of course, the moving iron armature.

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

(*Continued from page 196*).

23. *Mercurial Compensator for Barometric Error.*

As long ago as 1831, Robinson, of Armagh, showed how the barometric error might be compensated by means of a mercurial barometer attached to the pendulum. The arrangement is shown diagrammatically in Fig. 27.

The barometer should lie in the plane of the vibration, because, although it may not be obvious to the eye, any want of symmetry about that plane is apt to disturb the motion. On the other hand, a want of symmetry *in* the plane of vibration does not interfere with the vibration, although it may offend the eye.

Consider graph A of Fig. 8. If the lower surface of the mercury be level with the c.g. of the pendulum, a small movement of that surface will scarcely affect the rate; the change of rate with rise or fall of the mercury is then due to the motion of the upper surface almost entirely, and that surface will be about 220 mm. below the axis of motion of a seconds pendulum.

Fig. 27.—Robinson's Compensator for *Barometric Error* (1831).

The addition of a weight at this level is about 70 per cent. as effective as if added at the mid-point, where 1/10 800 of the mass of the pendulum is required for 1 second per day increase of rate. Consequently, a mass of mercury equal to 1/7500 of that of the pendulum must be shifted from the bottom of the column to the top to raise the rate by 1 s/d.

For pendulum G1, this mass is 0.68 gram, which would be that of 1in. of a mercury column of 1.6 mm. diameter. This gives the bore of tube required to correct a barometric error of 0.5 s/d. per inch of barometer, which corresponds to 1.0 s/d. per inch rise of the upper level if the bore be the same at top and bottom, for then the rise or fall of either surface is only half the change of barometer.

Referring again to Fig. 8, we see that if the barometer tube be fixed lower down on the rod the effectiveness of the motion of the upper surface is increased and at the same time the corresponding movement at the bottom also assists the compensation, making the compensation obtained greater.

On the other hand, if the tube be higher up, the effectiveness of the upper end is reduced and that of the lower end is negative, so that the total compensating effect is reduced, and becomes zero when the two surfaces are at equal distances from the mid-point of the pendulum.

Thus, if the bore of the tube lie within reasonable limits, it is possible to adjust the amount of compensation by sliding the whole thing up or down the rod, until it is found by trial to give the required result.

24. *Aneroid Compensator for Barometric Error.*

The aneroid chambers used for barographs have also been adapted by Riefler and others to form a compensator for the barometric error, as shown in Fig. 28. They carry a weight which they lift when the barometer goes down, and lower when it goes up. With the twin rod pendulum, the arrangement fits neatly between the rods, but with the usual pendulum it must be fixed to a bracket at one side, keeping it symmetrical about the plane of vibration.

Graph C of Fig. 8 shows how the movement of the mass affects the rate, and that it does so to a greater extent the higher it is. Thus, the higher the compensator can be fixed on the pendulum, the smaller is the mass required for a given degree of compensation.

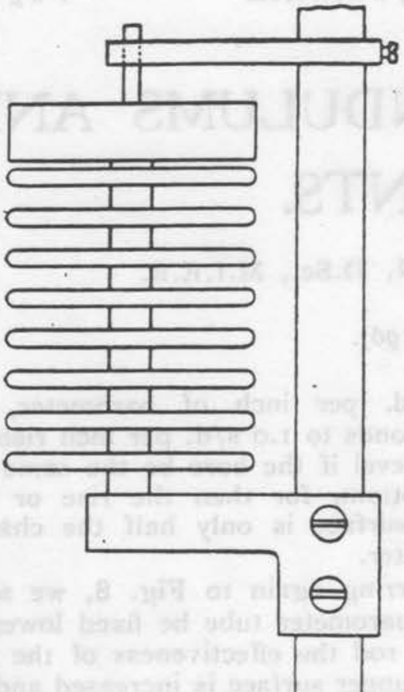


FIG. 28.—*Aneroid Compensator for Barometric Error.*

A compensator of this kind has been made for pendulum R3, whose mass is 5,800 grams, with the weight about 83 mm. below the axis. Graph C of Fig. 8 shows us that at this level a mass of 1/10 800 of that of the pendulum will alter the rate by 3.4 10^{-3} s/d. by moving 1 mm. The chambers actually move the mass 0.4 mm. for each 1in. change of barometer; thus the mass to be placed on the top to correct a barometric error of 0.4 s/d. per inch is:—

$$(5800 \text{ grams}/10\ 800) \times (1000/3.4) \times (0.4/0.4) = 154 \text{ grams} = 5.5 \text{ oz.}$$

For the ordinary single rod pendulum, with which the compensator has to be placed to one side to an extent not negligibly small compared with the distance below the axis, the equations of section 9 do not hold accurately, for they assume that the mass is in the centre line of the pendulum. But they will give an approximate result, and in any case the final adjustment would have to be made by trial.

Such adjustment may be made either by altering the level at which the compensator is fixed, or by adjusting the mass it carries.

The results of some tests made with this compensator on pendulum R3 are shown in Fig. 29, weights of 0, 100 and 200 grams being tried.

The tests were not very satisfactory, for not only has the pendulum an appreciable temperature variation when run in a sealed tank (it was designed for running in the open), but the aneroid appears to add very considerably to that variation.

With either weight on the aneroid top, but not with zero weight, the rate went up by a very large amount when the air pressure was reduced below 725 mm. The cause for this has not yet been definitely ascertained, but it is suspected that it is due to the rocking of the weight with the spring of the aneroid, as the steady pin shown on Fig. 28 was omitted.* It was not due to the pendulum striking a loose wire or other obstruction when the amplitude goes up with the fall of air pressure, and it is difficult to believe that so large an effect could be produced by any alteration of level brought about by a change of the inclination of the aneroid, which did not lie quite truly.

With the 200 gram weight, the change of rate from 750 mm. to 600 mm. is 14.6 s/d., which is 16 per cent. of the 93 s/d. by which the addition of the weight altered the rate at 750 mm. A downward movement of the weight of as much as 12 mm. would be necessary to increase the rate by 14.6 s/d.

It is intended to repeat this test under more favourable conditions.

Another compensating device, due to W. Ellis, has been employed at Greenwich Observatory. A magnet attached to the bottom of the pendulum is attracted by another one held below it by means of a lever whose position is controlled by a float in a mercurial barometer.†

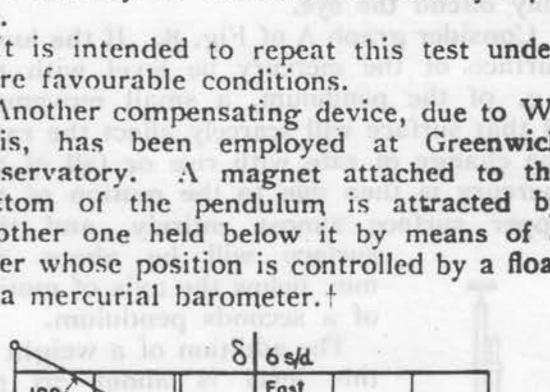


FIG. 29.—*Test results of Aneroid Compensator attached to Pendulum R.3.*

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* Further tests with a steady pin do not show this effect.

† An illustration of this device can be found, for example, in F. J. Britten's "Watch and Clock Makers' Handbook," p. 53. (Spon, 1922).

When the barometer rises, the lower magnet is raised nearer to that on the pendulum; their attraction is increased and the rate is made greater. By suitable adjustments, this increase of rate can be made approximately the same as the decrease due to the barometric change.

No barometric compensator can allow for changes of humidity.

V. Temperature Errors.

25. *List of Temperature Errors.*

A change of temperature produces a large family of disturbing factors; it alters the dimensions of every part, modifies the density of the atmosphere if that be kept at constant pressure, varies the viscosity of the air and also the friction at the pallets and pivots, changes the elasticity of the suspension spring and of the driving spring when there is such, and it may make an appreciable difference in the timing of the escapement forces because of differential expansion.

We thus have the following errors due to a rise of temperature :—

Decreasing the Rate.

(*a*) Expansion of the rod.

(*b*) Expansion of the suspension spring.

(*c*) Reduction of the elasticity of the suspension spring.

Increasing the Rate.

(*d*) Decrease of the buoyancy of the air. Absent with sealed pendulums.

(*e*) Decrease of the inertia of the air. Absent with sealed pendulums.

Increasing or Decreasing the Rate according to Circumstances.

(*f*) Expansion of bob. Increases the rate if bob be supported below a certain point slightly under its c.g.; decreases the rate if supported above that point.

(*g*) Increase of amplitude due to reduction of air resistance, causing circular and other indirect errors, decreasing the rate with unsealed pendulums.

With sealed pendulums, where there is no change of air density but only of its viscosity, the amplitude falls with rise of temperature and the rate is increased.

(*h*) Alteration of friction of pallets and pivots, causing both direct and indirect errors.

(*i*) Alteration in the driving forces due to change in the elasticity of the driving spring, or of friction in the driving train, causing direct and indirect errors.

TABLE III.

Estimated Temperature Errors of Pendulum G1.

Cause.	Drop.	Deviation Δ		δ/t.		Location of Errors.		
	10—8 mm./°C	10—6	s/d.	10—9 per °C.	10—3 s/d per °C	Top.	Distrib.	Bottom.
1	2	3	4	5	6	7	8	9
Expansions—								
Spring...	108			− 54	− 4.6	− 4.6		
Head	288			− 143	−12.4	−12.4		
Rod	1070			− 533	−45.9		−45.9	
Bob (Lift)	1190			+ 593	+51.1			+51.1
Dimensions		−2810	−242	− 66	− 5.7			− 5.7
Total of above	276			− 203	−17.5	− 17	−46	+45.4
Spring weakening ...		+7000	+600	−2000	−173	−173		
Air buoyancy and inertia		− 174	− 15	+ 610	+53		*	+53
Circular error Θ = 100′ ...		− 53	− 4.6	− 133	−11.5		†	−11.5
Total, excluding escapement errors				−1730	−149	−190	−46	+87

* Absent with sealed pendulum. † Smaller and + ve with sealed pendulum.

(j) Alteration in the timing of the escapement forces due to changes in the relative positions of the driving and driven elements brought about by differences in the expansions of the parts which locate them.

Errors (a) to (f), which belong to the pendulum, can be estimated more or less closely when the necessary data is available; the remaining errors depend mainly upon the escapement and can only be estimated with certain simple types of escapement.

Table III. gives the calculated errors for pendulum G1, and the following sections explain how the figures are obtained.

(To be continued.)

The Editor, "Horological Journal."
Dear Sir,

I am deeply indebted for your lengthy and practical suggestions in the February number. Please accept my best thanks.

A Canadian Reader.

Practical Column.

By J. W. PLAYER.

"These little incidents will occur even in the best regulated workshops."

This column is published with the idea of giving practical assistance to watch repairers and others who seek a solution of any difficulty they have to contend with, whilst at the same time the answers to questions will be found of great interest and help to other readers. All questions should be addressed to The Editor, Practical Column, Horological Journal, Fulwood House, High Holborn, London, W.C.1, by the 7th of the month for reply in the following month's Journal.

THE absence of any specific queries enables me to continue my remarks upon the topics introduced by D.T., of County Derry, by dealing briefly with some of the small tools in the outfit of a watch repairer. I have already touched upon screwdrivers and files and think that drills should have their turn.

There is quite a variety of these indispensables, but for the moment I will speak only of the ordinary arrow-headed drill as used with the bow, the other types for cutting in one direction only (as in a lathe) I will deal with at a later date, as they will require to be illustrated with sketches.

To be able to make a good drill is a most useful accomplishment and is one of the earliest lessons the apprentice should learn. I say should—too many never do, if what they call drills are to be taken as evidence.

The usual practice is to make a number or set from, say $\frac{1}{16}$in. downwards, and it is a good plan, as I have already pointed out, to number your drills according to the holes in the screw plate; for example, No. 7 drill should make a hole of the size to be followed by 7 tap, and so on and so forth.

The larger drills are better made in one piece with the stock; worn out round files are excellent for this purpose. A ferrule can be driven on to the shank and the other end softened and filed down to about halt the required size of the drill, then, resting the end of this reduced part on the edge of the square stake of the vice, beat it out with a flat-faced hammer to a fan shape. The object of this is to get clearance, that is to say, to make the cutting head larger than the shank, otherwise there is no room for the cuttings at the back of the head and the drill will stick; incidentally, also, it will toughen the cutting part. This beating out requires some practice to become adept at shaping the head really well and without cracking the steel. What one should aim at is to make the flattening taper off as long as possible to a thin edge so that to shape it up to a point requires only a little more than bevelling off the burrs, so to speak.

Too long or too short a point should be avoided, the two cutting edges should make an angle at the point of 45° to 50°, and the angle made by the whetting bevels should be about 30° to 35°.

After hardening, temper down to a deep straw for brass and give the edges a final whet up with an Arkansas slip, resting the stone on the bench and whetting each facet as you would a graver. It requires practice to acquire the skill to whet up these facets equally and keep the edges parallel and the point true, but of course it is not of much use to have a drill whetted to look well if the facets don't make true cutting edges culminating in a point. The purpose of the drill should not be lost sight of in the effort after symmetry.

The main thing to watch in these drills is the condition of the point, if that is worn down the drill is not likely to start accurately nor will it cut efficiently. Another thing is, if the point is out of centre, the hole it makes will be a little larger than was originally intended.

hour hand and the coloured disc, this masking disc having an aperture adjacent the end of the hand. In a further modification, Fig. 3, the dial *f* is illuminated by a lamp *x* at the rear, which lamp also lights up the coloured disc, the latter being of smaller diameter than the dial so that the ends of the hands show against the dial *f*. Moving with the hour hand *g* and having a hole *p* is a masking

FIG. 4.

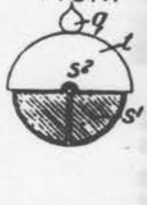


FIG. 12.

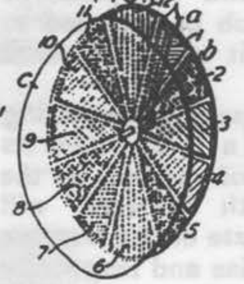


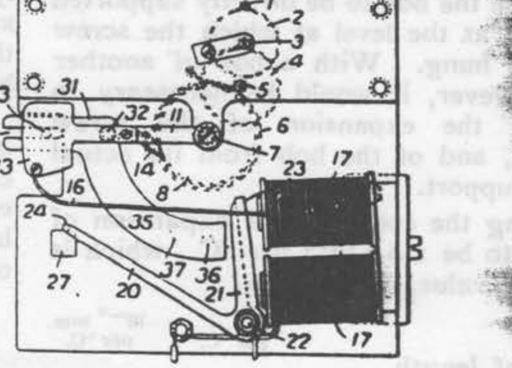
FIG. 3.

disc situated between the hand and the coloured disc. In a further modification, Fig. 4, a disc rigid with the hour hand *q* has one half s^1 of one colour and the other half s^2 of another colour, so that as the disc moves behind the masking semi-disc *t* the proportion of the two colours shown is an indication of the position of the hour hand. Still further modifications are stated in which (1) the axes of the coloured and masking discs are separate from, but geared to, the hour hand spindle, (2) four colours are shown in the same succession each hour, and (3) the colour may be changed rapidly at the end of each period.

305,327. Electric clocks. BOLD, H., 365, Liverpool Street, Seedley, near Manchester. Dec. 10, 1927, No. 33421. [Class 139.]

In an electric clock in which a weighted driving arm, pivoted on the minute wheel arbor is re-energised electromagnetically, indicator means connected to the weighted arm are provided to show when the electromagnetic force is insufficient to raise the weight. The driving arm 11, weighted at 13, is pivoted at 7 on the minute wheel arbor and carries a pawl 14 driving the ratchet wheel 8 rigid with the arbor 7 and connected by gearing 1 . . . 6 with an escapement which is controlled by a pendulum or by a balance. The weight is connected

FIG 1.



electrically in series by a wire 16 with the magnet windings 17 and contacts, to close the magnet circuit, with a surface 24 on a bell-crank 20, 21 pivoted at 22 and carrying the armature 23. A stop 27 limits the downward movement of the arm 20. When the weighted lever sinks the weight 13 contacts with the surface 24 and the magnet rocks the bell-crank 20, 21 clockwise so that the weighted lever is kicked upwards by the surface 24. A two-armed indicator member 31, 35 pivoted at 32 to the fixed frame of the clock is connected by a pin and slot 33, 34 to the weight 13 so that when the arm 11 is in its lowest position a red dot 37 on the indicator member shows through a hole 36 in the front plate of the clock and through a hole in the dial. In a modification the bell-crank carrying the armature is pivoted at one end.

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

(*Continued from page 218*).

26. *Expansion of Rod, Spring, etc.*

The coefficient of expansion of steel is about 12 millionths per degree Centigrade. With the spring, rod and bob made of steel, the temperature coefficient of the rate errors (*a*) and (*b*) would thus be -6×10^{-6} per °C., or -0.52 s/d. per °C.

With a rod and spring of invar (alloy steel with 0.1 per cent. carbon, 36 per cent. nickel and 0.5 per cent. or less of manganese) these figures would be about 12 times smaller.

As a rule, the pendulum is made of several metals, some of the parts being so arranged that their expansion raises the bob and annuls the drop due to the others. Step by step calculations must then be made.

Thus the pendulum G1 has a steel spring 18 mm. long, of which only half counts as part of the length of the pendulum, but the whole must be allowed for when estimating the relative displacement of parts carried by the pendulum and those fixed on the

frame. The head by which the invar rod is attached to the spring adds another 24 mm. of steel, and then there are 1,070 mm. of invar down to the point at which the rating screw is supported.

Since the rating screw and bob are in this case of the same metal, steel, we need not worry about the expansion of the screw, but suppose the bob to be directly supported by the rod at the level at which the screw actually is hung. With a bob of another metal, however, it would be necessary to work out the expansion of the screw separately, and of the bob from its actual point of support.

Assuming the coefficient of expansion of the invar to be 1.0×10^{-6} per °C., which is an average value, we have:—

	per °C.	10^{-6} mm. per °C.
Increase of length of half spring...	$9 \times 12 \times 10^{-6}$	$= 108$
Increase of length of head ...	$24 \times 12 \times 10^{-6}$	$= 288$
Increase of length of rod	$1070 \times 1 \times 10^{-6}$	$= 1070$
Total		1466
		per °C.
Ratio to distance of c.g. of bob from axis (1004 mm.) ...	1.46×10^{-6}	
Rate temperature coefficient due to this	0.73×10^{-6}	
	$= 0.063$ s/d.	

It is obvious that if the correct figure for the particular invar rod be 1.1 or 0.9 instead of 1.0, or that for the steel be 11 instead of 12, our calculation will be very considerably out.

These coefficients do vary somewhat from specimen to specimen, and the resulting uncertainty makes any calculation of the temperature variation of the rate only roughly approximate.

As there are other uncertainties of at least equal magnitude, it would not be worth while determining the coefficient for each individual bar, a process requiring the resources of a national laboratory for it to be carried out with the necessary accuracy; it is easier and much more satisfactory to determine the rate temperature coefficient of the complete pendulum when running under exactly the required conditions, for this coefficient varies with these conditions to quite a large extent.

27. Expansion of Bob.

If the bob be supported at its centre of gravity, then its own expansion does not affect the position of that point, but it slows the rate by the increase of the local inertia due to its finite dimensions.

If supported below the c.g., the bob will be raised by the expansion of the portion lying between the c.g. and the point of support, whereas if the bob be attached to the rod at a higher level it will be lowered by its own expansion.

The bob can be made self-compensating by supporting it at such a level below its c.g. that the upward lift of the bob by the expansion of the length between will balance the effect on the rate of the increase of inertia due to the sidewise and lengthwise expansion of the whole bob.

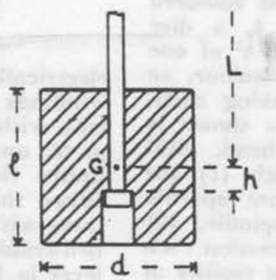


FIG. 30.—*Point of Support for Self-compensating Bob.*

In Fig. 30, let h be the distance of the point of support below the c.g.; let the coefficient of expansion of the bob be a_b and suppose that the distance of the point of support down from the axis of motion remains invariable, then:—

Local inertia

$$M\,(l^2/12 + d^2/16) \qquad (27.01)$$

Increase of same per unit temperature rise

$$2a_b M\,(l^2/12 + d^2/16) \qquad (27.02)$$

Effect of this on the rate

$$-a_b\,(l^2/12 + d^2/16) \div L^2 \qquad (27.03)$$

Lift of c.g. per unit temperature rise

$$a_b h \qquad (27.04)$$

Effect of this on the rate

$$\tfrac{1}{2} a_b h / L \qquad (27.05)$$

If the total effect on the rate is to be zero, then:—

$$\tfrac{1}{2} a_b h / L = a_b\,(l^2/12 + d^2/16)/L^2 \qquad (27.06)$$

or,

$$h = 2\,(l^2/12 + d^2/16)/L \qquad (27.07)$$

For pendulum G1 $(l^2/12 + d^2/16) = 5700$ mm.² and $L = 1004$ mm. (see Table I.). Hence $h = 11.4$ mm. or 0.45in.

For the cylinder of minimum surface, where $l = d$, $h = (7/24)\,d^2/L$.

When the bob is not supported at the self-compensating point, the upward lift of its c.g. by its own expansion must be deducted from the drop due to the expansion of the rod, etc., and the effect of the local inertia allowed for separately.

Let a_L be the net coefficient of expansion of L, after taking account of the spring, head, rod, bob, etc. In the expression for the deviation due to the local inertia of the bob, namely,

$$\Delta_b = -\tfrac{1}{2}\,(l^2/12 + d^2/16)/L^2 \quad (27.08)$$

the numerator has the coefficient $2a_b$ and the denominator one of $2a_L$, giving a total coefficient of $2\,(a_b - a_L)$.

The corresponding temperature error is:—

$$\delta_b/dt = -(a_b - a_L)\,(l^2/12 + d^2/16)/L^2 \quad (27.09)$$

After allowing for the expansion of the rating screw, the virtual attachment of the bob of pendulum G1 to its rod is 99 mm. below the c.g. of the bob. The expansion of this 99 mm. is 1190×10^{-6} mm. per °C., which must be deducted from the 1,466 already found for the rod, head and spring, leaving a net amount of 276×10^{-6} mm. per °C. This corresponds to a value of $276 \times 10^{-6} \div 1004$, or 0.275×10^{-6} per °C. for a_L, or 11.7 for $(a_b - a_L)$.

Putting this and the values we already know in equation (27.09), we find that the temperature error due to the local inertia of the bob is -0.066×10^{-6} per °C., or -0.0057 s/d. per °C.

It will be observed that the upward lift due to the expansion of the bob has wiped out about 80 per cent. of the drop due to that of the rod, etc. If the point of virtual attachment had been at the bottom, they would almost exactly have balanced one another. This is one common way of obtaining compensation.

28. *Weakening of Spring.*

A rise of temperature weakens the spring in addition to increasing its dimensions. In the expression for λ (equation (10.07)), (l/c), M and g are independent of the temperature, but b and c increase with rise of temperature whilst E falls. For a steel spring, the corresponding coefficients are about 12 and -240 millionths per degree Centigrade.

Since λ is proportional to $1 \div \sqrt{(bcE)}$, its temperature coefficient is:—

$$a_\gamma = -\tfrac{1}{2}\,(a_{el} + 2a_{ex}) = -\tfrac{1}{2}a_{el} - a_{ex} \quad (28.01)$$
$$= +108 \times 10^{-6} \text{ per °C. for a steel spring} \quad (28.02)$$

This coefficient is the proportional increase of λ per unit rise of temperature; that for Δ_s is different in the ratio $(dA/A) \div (d\lambda/\lambda)$. Hence the temperature error of the rate caused by the weakening of the spring is:—

$$\delta_s/dt = a_\lambda \times \{\,(dA/A) \div (d\lambda/\lambda)\,\} \times \Delta_s \quad (28.03)$$
$$= \{\,a\lambda dA \div (d\lambda/\lambda)\,\} \times (l/L) \quad (28.04)$$

since $\Delta_s = A\,(l/L)$.

Use may be made of the graphs of Fig. 13, but for greater convenience Fig. 31 has been drawn to give the value of the quantity within the large brackets of equation (28.04), taking the value given in equation (28.02) for a_λ. The temperature error due to the spring is then obtained by calculating λ from equation (10.07), reading off the corresponding ordinate on Fig. 31, and multiplying the result by the value of (l/L). The error is always negative whether the deviation be positive or negative.

Thus, in section 12 we found that for the spring of pendulum G1, $\lambda = 1.03$ and $(l/L) = 0.0184$. Fig. 31 gives -9.0 s/d. per °C. for this value of λ; multiplied by 0.0184 this gives -0.17 s/d. per °C. as the temperature error due to the weakening of the spring.

This error is very large, being more than 12 times that caused by the expansions; this is the result of employing such a stiff spring for the reasons already given. The 8-mil spring would give -0.031. The spring of pendulum R3 gives -0.054.

As we saw in section 11, no appreciable reduction of this error is possible by making the spring of greater length than corresponds to $\lambda = 3$. Beyond that point the decrease of the ordinate of Fig. 31 is

FIG. 31.—*Temperature Error Coefficient due to Steel Suspension Spring.*

practically wiped out by the increase of (l/L).

For accurate clocks, the spring should be made of the alloy Elinvar already mentioned in section 11. For this alloy, the elasticity temperature coefficient varies for different samples between the limits ± 50 millionths per °C., and its coefficient of expansion is about 6 in the same units. Thus a_λ lies between +20 and −30 millionths per °C. The worst specimens give a temperature error five times smaller than with steel, and the best ones give zero error, or a positive one which is to be preferred.

29. *Temperature Error of Atmospheric Deviation.*

At a temperature of 15°C., the density of air falls by 1/288 of its value for 1°C. rise of temperature. The temperature coefficient of the air buoyancy and inertia deviations is thus 1/288, or 3.5×10^{-3} per °C., when the air is permitted to expand at constant pressure.

Assuming the sum of these two deviations to be 15 s/d., the corresponding temperature error will be +0.053 s/d. per °C. This is more than sufficient to wipe out the effect of the expansions if the whole pendulum be made of invar, and it forms a very important part of the compensation of the ordinary unsealed pendulum. It is equivalent to an upward lift of 1.2 microns for a seconds pendulum, which is that which would be given by the expansion of 100 mm. of steel or 65 mm. of brass.

When the pendulum is sealed up hermetically, the density of the gas surrounding it is not affected by rise of temperature, but only its pressure. The consequence is that this compensating effect is absent, and the compensator must be made about 65 mm. longer (if of brass) than would be necessary for the same pendulum run in an unsealed case. The exact amount depends, of course, upon the magnitude of the air inertia deviation for the pendulum concerned.

30. *Other Temperature Errors.*

When the change of amplitude is known, it is easy to calculate the error due to the change of the circular deviation with any running amplitude, but it is only in exceptional cases that it is possible to make even a rough estimate of the other effects of change of amplitude, or of those which arise from the alteration in the escapement setting due to differential expansions.

All these effects depend upon the design and adjustment of the escapement, and it is only with certain simple types of escapement that there is any possibility of getting all the data needed for such an estimate.

The author's link escapement is one such type. The clock of the University of Bristol has this escapement and a pendulum identical with R3. For the sake of appearance, the baseplate is of bronze, but the pendulum rod and the link are of invar. As a result of the excess of the lowering of the lever bearing over the expansion of the rod and link, there is a temperature error as large as −0.16 s/d. per °C. With a cast iron base this figure would have been −0.092.

In this particular case, the pendulum is run at such an amplitude that the escapement errors and the circular error produced by a change of amplitude almost exactly neutralise one another, but in the general case the escapement errors may either be of the same or opposite sign as the circular error, and they may be larger or smaller.

Fig. 32 shows the estimated amounts of the circular error due to rise of temperature when running in an unsealed case, on the assumption that the amplitude rises by 3.4×10^{-3} of its own value for 1°C. rise of temperature. This is about the value found for pendulum R3 from Fig. 23. If the pendulum were run in a sealed tank, the change of amplitude would be much less and would be of opposite sign, for a rise of tempera-

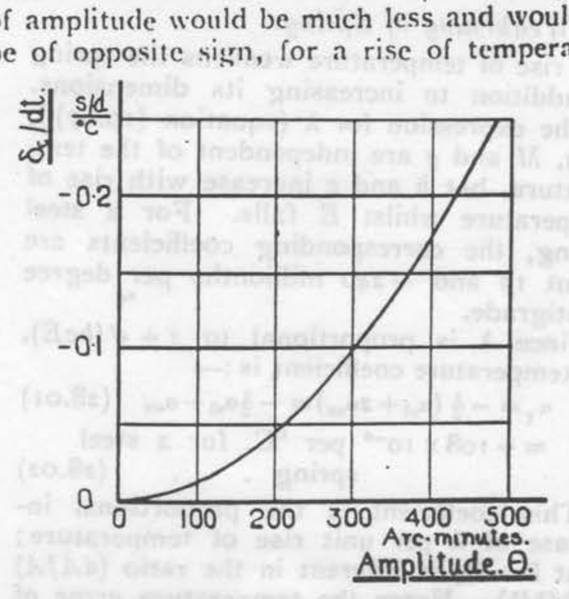


FIG. 32.—*Estimated Circular-Temperature Error for Pendulum G1.*

ture would not then affect the eddy resistance, but only the skin friction which would be only a part of the total.

31. *Temperature Compensation.*

Temperature compensation is obtained by supporting the bob in such a manner that it is raised by the expansion of certain parts when the temperature rises, to an extent sufficient to annul the other temperature errors. Such an arrangement is shown in Fig. 33.

FIG. 33.—*Compensator for Temperature Error.*

The compensator is made of some material of high expansibility, such as brass or zinc, and the compensation is due to the excess over that of the extra length of rod which it entails. The net coefficient for the compensator is obviously the difference between its own one and that of the rod.

Given these coefficients of expansion, and knowing that a lift of 23 microns ,is required for an increase of one second per day with a seconds pendulum, it is easy to calculate the length of compensator needed to correct any specified temperature error. Table IV. gives the necessary data for the more usual materials.

TABLE IV.

CONSTANTS FOR TEMPERATURE COMPENSATORS.

Materials.	Millionths per °C.			mm. for 1 s/d per °C.
	a_1	a_2	$a_1 - a_2$	
Mercury—	182			
Invar... ...		2×1	180	128
Steel		2×12	158	145
Zinc—	26			
Invar... ...		1	25	920
Steel		12	14	1640
Fused Silica...		0.42	25.6	900
Beech ...		3	23	1000
Brass—	19			
Invar... ...		1	18	1280
Steel		12	7	3300
Fused Silica...		0.42	18.6	1240
Type-metal—	19			
Invar... ...		1	18	1280
Steel—	12			
Invar... ...		1	11	2090
Fused Silica...		0.42	11.6	1930

The right length of compensator is not that which will keep the c.g. of the bob at a constant level, but that which will cause an error equal and opposite to the sum of all the other errors, including the expansion of the rod down to the *top* of the compensator.

An estimate must therefore be made of these errors in the manner just explained. For example, in Table III. we estimated the temperature errors of pendulum G1 without compensator at −0.149 s/d. per °C. To correct this by a brass-invar compensator would take a length of 0.149 × 1280, or 191mm. Using a zinc compensator we should need 137mm.

But the uncertainties of such calculations are very considerable because of the doubt as to the correct values of the coefficients of expansion of the various pieces, and as to that of the temperature coefficient of the elasticity of the spring. When a really accurate compensator is wanted, its correct length must be found by trial.

After fitting a compensator somewhat greater than is expected to be necessary, tests must be made to determine the temperature coefficient of the rate of the pendulum under conditions identical in all respects, including amplitude and setting of the escapement, with those under which the pendulum is to be run in use, for several of the errors vary with these conditions.

(*To be continued*).

❖

Messrs. J. B. Joyce & Co. Ltd., Whitchurch, Salop, have received instructions to supply a large Tower Clock striking the hours, with four dials, for The Lawrence Memorial Royal Military School, Nilgirio, India, and another similar one for Bathhurst, Gambia. They have recently added the Westminster Chime Quarters to the clock at Nantwich Parish Church, and thoroughly restored and added an additional dial 8ft. diameter to the clock at St. John's Church, Coventry.

❖

Kew Watch Trials.

The latest dates for receiving watches at the British Horological Institute (35, Northampton Square, London, E.C.1), for the next National Physical Laboratory Trials are Wednesday, 15th and Thursday, 30th May, 1929.

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

(Continued from page 240).

SUPPOSE that we are using a brass compensator, and that these tests show that the pendulum is over compensated leaving a positive temperature error (δ/dt). If we cut a length off the compensator equal to 3,300mm. × (δ/dt), and replace it by an equal length of steel, we should reduce the temperature compensation by the right amount. With a zinc compensator, the amount to be removed would be exactly half the above. Fig. 34 shows such a two-piece compensator.

As the workshop process of adjusting the lengths can be carried out quite easily to a fraction of a mm., there is no difficulty, so far as that part of the business is concerned, in reducing the temperature coefficient of the rate below 1/10,000 s/d. per °C. The limit of accuracy with which the compensator can be adjusted is thus the limit with which the above test can be carried out, and that in turn will be that at which other irregularities predominate over the temperature effects. Obviously nothing is to be gained by getting the compensator more accurate than that if we could do so, for the same irregularities which affect our test will also affect the time-keeping of the clock in use.

Since it is always possible to employ a greater range of temperature for the test than will occur in use, there should be no great difficulty in making the temperature errors too small to be detected in comparison with other uncontrollable irregularities.

As already pointed out in section 27, when the bob is more expansible than the rod, a considerable amount of compensation can be obtained by supporting it below its c.g.

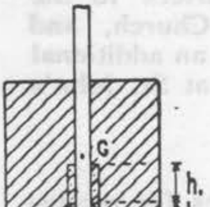


FIG. 34.—*Two-piece Temperature Compensator.*

Many of the old compensators make use of this fact. The oldest of all, and the most effective, is the mercury bob invented in 1722 by George Graham (1673-1751), in which the mercury is carried by a glass or iron container attached to the rod.

With bob compensation, it is more convenient to estimate the expansion of the rod (when the top of the container is attached to the rod, as is usual with an iron container, the expansion of the container down to the bottom of the mercury is to be reckoned in with that of the rod) to the *bottom* of the bob, when estimating the temperature errors, and then the correction is calculated from the expansion of the lower half of the bob.

In millionths per degree centigrade, the coefficient of cubical expansion of mercury is 182, but the sidewise expansion of the container causes an apparent reduction of 2 × 12, leaving an effective value of 158.

Zinc bobs with wooden or iron rods have often been employed in this way. The wooden rod has a very small thermal expansibility, but it always shows changes in length with changes in the humidity of the surrounding atmosphere. In some old records of the author's for a pendulum with a wooden rod, there is a very marked difference between the winter season when the air has a low relative humidity because of the artificial heating of the college building, and the summer season when the relative humidity in Bristol is very high. When the fires were started towards the end of September, the rate gradually rose for some weeks as the rod dried out, and it gradually fell again after the fires ceased in the spring.

The first solid compensator was the gridiron pendulum invented in 1726 by John Harrison (1693-1776), who got over the difficulty of dealing with the great length of compensator required with the materials at his disposal (iron and brass) by dividing the length into several parts, alternately iron and brass, placed sde by side.

The same principle was applied later by Dent in a simpler form, by using concentric tubes of iron and zinc. Dent's arrangement is to be found on the pendulum of "Big Ben."

The invention of the nickel steel alloy "invar," has made all these old methods of compensation obsolete, and as they are fully described in hundreds of textbooks there is no need to do so in detail here.

An interesting example of the slowness with which exact information travelled through a trade a century or so ago may be quoted. The author has a clock made nearly 100 years ago by a maker of considerable repute (Dobbie of Falkirk). But although over a hundred years had elapsed since Harrison's invention, the brass in the "compensator" expands downwards, and the iron upwards!

32. *Location of Temperature Compensator.*

Chiefly because it is more easily fixed there than elsewhere, the temperature compensator is generally placed at the foot of the pendulum as in Figs. 33 and 34. But sometimes it takes the form of a ring flush with the outside of the bob instead of a tube at its centre, so as to enable it more quickly to follow the changes of air temperature.

In the last three columns of Table III., the errors are collected into groups in accordance with the location of their causes. Of the total estimated error, −0.190 s/d. per °C. occurs at the top of the pendulum and +0.087 at the bottom; a compensator added to the bottom to make the total error zero would increase the latter figure to +0.236.

But a positive error at the bottom will only annul a negative error at the top when the temperatures are alike at these two places, and there is a probability, rather than a mere possibility, that such will not be the case.

Suppose the top to be 1°C. above the mean temperature and the bottom to be the same amount below. With this particular pendulum, plus a compensator added below, the former would cause an error of −0.190 s/d., and the latter one of −0.236, making a total of −0.426 s/d. This is much bigger than any single item in the table, and indeed greater than the sum of all the items of one sign. Thus very serious errors may be caused by quite small differences of temperature between the top and bottom levels even when the compensator is quite accurate for uniform temperature.

It is true that things are somewhat exaggerated in this pendulum by the stiff suspension spring, but similar results not so very much smaller in amount will hold for any pendulum with a steel spring. Sealing the case does not affect this question, for although the air buoyancy and inertia errors are thereby wiped out, the error is only transferred to the compensator at the same level.

The ideal arrangement would be to make the bottom and top parts each have a positive error just sufficient to annul half that due to the expansion of the rod; a gradual change of temperature from the bottom to the top would then affect the rate very slightly if at all.

Leaving out the lift due to the bob, the sum of the items in column 9 of Table III. is +36; adding half the rod error, namely, ½ of −46, we get +13. If the bob be supported 25mm., or 1.0in. *above* its c.g., the temperature error due to the drop caused by its expansion will be −13, and we have attained our desire at this end of the pendulum.

If the pendulum be run in a sealed case we would have −23−6+ a small positive allowance for the circular error, which we shall guess at +4, giving a total of −25; the bob should now be supported 48mm., or 1.90in., *below* its c.g.

In either case, we are left with −190 −23, or −213 ms/d. per °C. to be compensated at the top of the pendulum, which will require 273mm. (10.8in.) of a brass-invar compensator, or 196 mm. (7.7in.) of a zinc-invar one. These amounts are inconveniently large, but with a suspension spring of elinvar, or even with a thin spring of steel, they will be reduced to lengths which can be quite readily dealt with.

This desirability of getting the compensator at the top of the pendulum emphasises the value of elinvar for the suspension spring, for that alloy not only reduces the total amount of compensation needed, but it brings it within the range which is easily dealt with at the top.

Fig. 35 shows a simple way of arranging a compensator at the top end of a flat rod. The head for taking the suspension spring is of invar and has two invar cheeks projecting down some way on each side of the invar rod. The cheeks and rod are slotted to take a slip of brass, or zinc, whose bottom end is supported in the bottom of the cheek slots, and whose top carries the rod.

The slots should be a little longer than the slip, which should fit sufficiently well

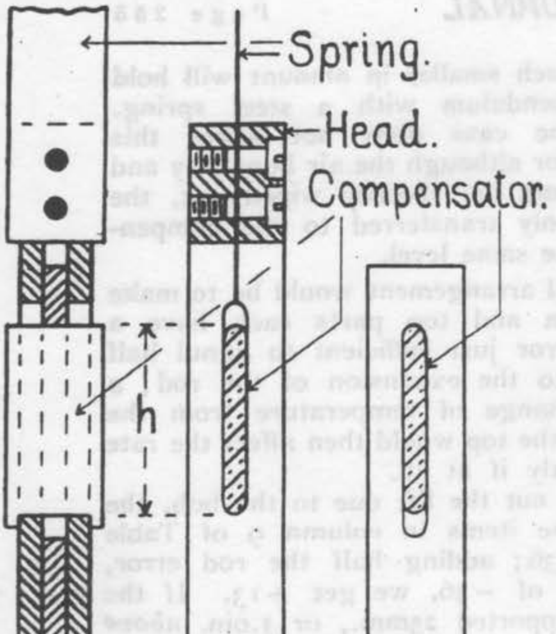


FIG. 35.—*High Level Temperature Compensator.*

at the sides to make the whole thing oscillate as one piece, but not so tightly as to prevent the weight of the pendulum from pulling everything home lengthwise.

As it is easier to make a new slip than to fill up one end of the slot in the rod, the first trial compensator should be made shorter than is expected to be needed, or the first test for the temperature compensation can be made without any compensator. When the temperature error has been found by trial, the correct length of compensator can be calculated from the data given, a new slip cut, and the slots lengthened to take it.

ERRATA.

P. 240, alongside Fig. 33, insert the words "of its expansion" between "excess" and "over."

✣

A new Booklet of the British Horological Institute has just been issued giving particulars of Membership, list of Members, etc. Any Members or others interested may have a copy on application to the Secretary at the Institute, 35, Northampton Square, London, E.C.1.

Practical Column.

By J. W. PLAYER.

"These little incidents will occur even in the best regulated workshops."

This column is published with the idea of giving practical assistance to watch repairers and others who seek a solution of any difficulty they have to contend with, whilst at the same time the answers to questions will be found of great interest and help to other readers. All questions should be addressed to The Editor, Practical Column, Horological Journal, Fulwood House, High Holborn, London, W.C.1, by the 7th of the month for reply in the following month's Journal.

R. E. G. NEWMACHER writes:—I have an 8-day grandfather clock under repair which has so far beaten me to get in proper order. During the first four days it goes all right, but when the weights are little more than half down it stops; this was the fault when I got it to repair. When I took it down I found the centre pivot at the back very much cut and the pallets cut and other holes wide. I have now reduced centre pivot and polished it, re-faced pallets and polished them bright with red stuff, bushed all holes that were wide and cleaned all the clock, but still it stops when half down. It is strange it goes when wound up. I thought there would be no difference as regards the pull of the weight whether it was up or half up. What would you suggest I can do? It has a recoil escapement.

Answer.—R. E. G. is quite right in thinking that the pull of the weight does not vary whatever its position may be, but influences may come between and interfere with the full exercise of the pull. For instance, the gut may rub somewhere or it may bind in the barrel grooves, either because it is not the right size or from variation in thickness or because the grooves themselves are in bad condition; besides, many of these clocks have a date mechanism and show the phases of the moon, if this apparatus is not in free working order it may bind at some point and so diminish the effectiveness of the power.

I am inclined to think, however, that the true cause is the proximity of the weight to the pendulum. The weight and its cord in themselves constitute a pendulum of sorts, and as the weight falls this unofficial pendulum begins to approximate in length to that of the real pendulum. If and when this happens, and if also the weight is sus-

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

(*Continued from page 256*).

VII. DISTURBED HARMONIC VIBRATIONS.

33. *Disturbed Harmonic Vibrations.*

Very often, as with a pendulum, a vibration is almost a true harmonic one, but not quite so, because of some force which does not follow the required law of being exactly proportional to the displacement. Such a vibration may be termed a "disturbed harmonic vibration," and is most easily treated by looking upon it as a true harmonic vibration upon which are superimposed certain disturbing forces.

Such disturbing forces may be divided into three groups:—

(*a*) Discrepancy forces, being the differences between the actual controlling force and one strictly proportional to the displacement. Over a whole cycle, these forces neither impart energy to the vibration, nor abstract energy from it.

(*b*) Frictional forces, absorbing energy from the vibration.

(*c*) Escapement forces, which on the whole give energy to the vibration.

With a pendulum, we get forces in the first group because the restoring force is proportional to $\sin\theta$ instead of to θ. The corresponding deviation is known as a "circular" deviation, because it arises from the shape of the path of the c.g. of the pendulum. If the c.g. be made to move in a cycloidal arc, instead of in a circular one, there would be no discrepancy forces.

The circular deviation lies between 1 and 40 seconds per day, according to the amplitude, but the forces causing it are not subject to erratic variations and consequently are not a primary source of rate error. But as a result of changes of amplitude brought about by accidental changes in other forces, they are responsible for indirect errors which may amount to 1 s.d., and which may be far more important than the direct effects of such accidental changes.

With a balance wheel, or a torsional pendulum, as the factor $\sin\theta/\theta$ does not come in there is no true "circular" deviation, and it is commonly supposed that they are therefore free from errors due to change in amplitude. This supposition is entirely erroneous, for not only does it forget that the escapement deviations are modified by change of amplitude, both directly because the amplitude is a factor in their amount, and also indirectly because the timing of the escapement action varies with the amplitude, but it assumes that the material follows Hooke's law with absolute accuracy.

Now, with a pendulum swinging through 100 arc-minutes on each side of the centre, the maximum discrepancy force is only 1/7100 of the restoring force. It is doubtful whether there is any material whose stress-strain curve agrees with a straight line to this degree of accuracy, and it is certain that for most materials the discrepancy is much greater.

Consequently, elastically controlled vibrations do have discrepancy deviations and errors in their rate, which are almost certainly not less than the circular ones of the ordinary pendulum, and which have the additional disadvantages that there is no data available from which they might be calculated, and that there is no certainty that they are not subject to erratic changes.

The effects of air friction on the rate have already been discussed in section 21, and the effects of solid friction and of the escapement forces will appear as we proceed.

34. *Constant Force.*

If a constant force, F, act in the positive direction, the equation to the motion becomes:—

$$\omega_0{}^2x - F/M + d^2x/dt^2 = 0 \qquad (34.01)$$

or,

$$\omega_0{}^2\,(x - F/M\omega_0{}^2) + d^2x/dt^2 = 0 \qquad (34.02)$$

$$\omega_0{}^2\,(x - y) + d^2x/dt^2 = 0 \qquad (34.03)$$

where,

$$y = F/M\omega_0{}^2 = F/k \qquad (34.04)$$

This is a simple harmonic vibration of the original frequency, but with its centre

displaced by an amount y in the direction of the force F. Thus, the application of a constant force and a change of zero are synonymous terms.*

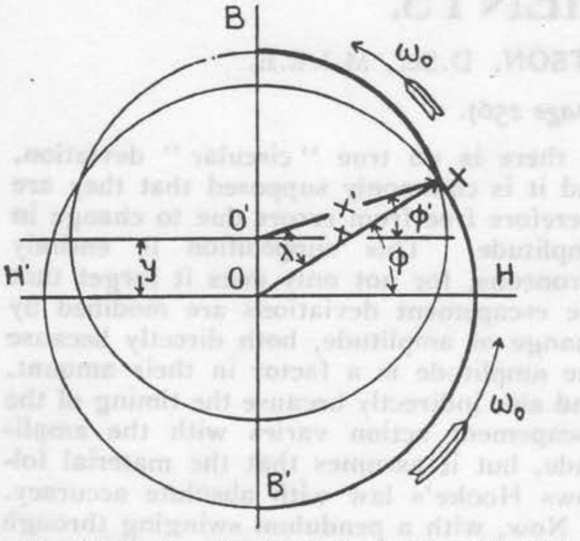


FIG. 36.—*Application of a Constant Force, or Change of Zero.*

The new vibration may thus be represented, as in Fig. 36, by a vector $O'X$, of constant length X', revolving with the original phase velocity ω_0 about a point O' in the reference line distant y from O. The vector OX, drawn from O, must consequently vary its length and phase velocity in such a manner that the end X moves uniformly round the circle having O' as its centre, instead of in one concentric with O.

The period is thus exactly the same whether the force is on or off, and its *presence* does not affect the energy of the vibration. But as $O'X$ is not, in general, either equal or parallel to OX, the *application* (or removal) of the force will cause a change of energy and a loss or gain of phase referred to the uniformly rotating vector. It causes no loss of phase at the ends of the swing, and no change of amplitude if it take place at the mid-point between O and O'.

We thus see that it is only by *changing* the force acting on the pendulum that energy can be given to, or taken from, the vibration, and that, except when it occurs at the end of the swing, each such *change* will cause a loss or gain of phase.

*** H. R. A. Mallock draws attention to this fact on page 509 of his paper on "Pendulum Clocks and their Errors." (Roy. Soc. Proc. Ser. A., Vol. 85, pp. 505-526, 1911), but fails to deduce the important connection between it and the energy given, or absorbed, which is stated below.**

If the application of the force be instantaneous, the change of the energy of the vibration, referred to the new centre, is also instantaneous, and does not wait until work can be done by the force acting through some distance. At first sight, this statement appears to contradict the law that energy cannot be instantaneously transferred from one place to another, or transformed from one kind to another, without the use of an infinite force or its equivalent.

But the stipulation that the force is to remain constant after it is once applied involves the postulate that the agent which exerts the force must itself possess a store of potential energy which can increase and decrease as the pendulum swings to and fro.

All that has been done by the application of the force is to label a portion of the original energy of the external agent as now belonging to the vibration, or *vice-versa*. There has been no physical transfer of energy, but only a book-keeping entry by which so much energy is credited to one account and debited to another.

This idea of adding to the total energy of the vibration is of the utmost value in simplifying the calculations of escapement errors, and is enormously superior to the older methods, which in the past have often led to erroneous conclusions.

Let the force be applied at the instant for which the diagram is drawn, when the amplitude is $OX = X$, and the phase is $HOX = \phi$. Immediately afterwards, the amplitude is $O'X = X'$, and the phase ϕ', both referred to the new centre O'.

From the triangle OXO' we have:—

$$X'^2 = X^2 + y^2 - 2yX \sin\phi \qquad (34{\cdot}05)$$

Hence the increase of the energy of the vibration is:—

$$W_0 = \tfrac{1}{2}kX'^2 - \tfrac{1}{2}kX^2 \qquad (34{\cdot}06)$$
$$= \tfrac{1}{2}k\,(y^2 - 2yX\sin\phi) = ky\,(\tfrac{1}{2}y - x) \qquad (34{\cdot}07)$$
$$\doteq -kyX\sin\phi \text{ if } y \text{ be small} \qquad (34{\cdot}08)$$
$$= -FX\sin\phi = -Fx \qquad (34{\cdot}09)$$

If the force be applied at the instant at which the displacement is $x = \tfrac{1}{2}y$, there would be no change of energy. Before that point there would be a gain and after it a loss. If the force be applied before that point and removed after it, there would be a gain of energy at both operations.

Observe that the *application* of a force towards the centre increases the total energy of the vibration whether the motion at that instant be inwards or outwards, in spite of the fact that in the latter case the force begins to abstract kinetic energy from the pendulum. Similarly, the application of a force away from the centre always diminishes the total energy.

Now, any action whatever which disturbs the motion of the pendulum may be regarded as a *series of applications* of small forces, of which any one dF will add energy to the vibration of an amount :—

$$dW_e = -dFX\sin\phi \quad . \quad (34.10)$$

and the total energy given by them all is obtained by adding all the little bits of energy together. Expressed in the usual way, that is :—

$$W_e = \int dW_e = -\int dFX\sin\phi \quad (34.11)$$

35. *Loss of Phase with the Application of a Force.*

Except at the very ends of the swing, the application of a force in the same way as the movement will always cause a loss of phase, and the application of one against the motion a gain of phase.

From the trigonometry of triangle OXO', we get :—

$$y/\sin\lambda = X'/\cos\phi \quad . \quad (35.01)$$

With the forces concerned in escapement action, y/X' is of the order of 10^{-3} or less. Consequently λ is a very small angle, X and X' are practically alike, as are also ϕ and ϕ'. We may therefore write :—

$$\lambda = \sin\lambda = (y/X)\cos\phi \quad \cdot \quad (35.02)$$
$$= (F/kX)\cos\phi \quad \cdot \quad (35.03)$$
$$= -(W_e/kX^2)\cot\phi \quad \cdot \quad (35.04)$$
$$= -\tfrac{1}{2}(W_e/W_v)\cot\phi \quad \cdot \quad (35.05)$$

Where,

$W_v = \frac{1}{2}kX^2$ = total energy of vibration (35.06)

$W_e = -FX\sin\phi$ = energy added to vibration by the application of the force (35.07).

Equation (35.03) applies when we have to deal with a definite force, but with escapement action the force has to be chosen to suit the selected phase of application because a definite amount of energy must be supplied to maintain the desired amplitude. One of the two next equations should then be employed.

Note especially that the loss of phase when a definite amount of energy has to be given is proportional to the ratio of that energy to the total energy of the vibration. The same applies to the direct rate errors caused by accidental changes in the escapement.

In the general case, where we have a series of applications of small forces, dF, the equation for the loss of phase becomes:

$$\lambda = \int d\lambda = \int (dF/kX)\cos\phi \quad (35.08)$$
$$= -\tfrac{1}{2}\int (dW_e/W_v)\cot\phi \quad (35.09)$$

For a complete cycle, we have to take all values of ϕ from 0 to 2π, and the fractional loss of phase is then $\lambda/2\pi$. The rate deviation, when the same action is repeated every cycle is thus :—

$$\Delta = -\lambda/2\pi$$
$$= -(1/2\pi)\int_0^{2\pi}(dF/kX)\cos\phi \quad . \quad (35.10)$$
$$= (1/4\pi)\int_0^{2\pi}(dW_e/W_v)\cot\phi \quad (35.11)$$

36. *Constant Frictional Force.*

Let there be a frictional force opposing the motion of constant amount F. It will reverse at the same instant as the motion does at the end of each swing; that is exactly when $\phi = 90^\circ$, $\cos\phi = 0$, and $x = X$.

But we have seen that a change of the force at the end of the swing cannot cause any deviation of the rate. Consequently, a constant frictional force will produce no rate deviation whatever its amount, and its accidental variations will cause no direct rate errors so long as it does not vary from point to point in a single swing.

But accidental changes in the friction do cause variations of amplitude, with the

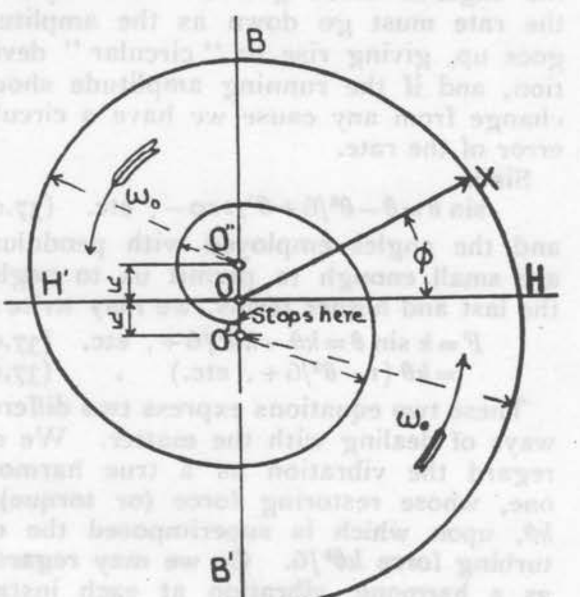


FIG. 37.—*Constant Frictional Force.*

result that they give rise to quite large indirect rate errors, by modifying the circular deviation and the deviation caused by the escapement forces, and by altering the timing of the escapement actions.

Fig. 37 shows an oscillation whose decay is brought about by a constant frictional force. The amplitude, measured from the true zero, gets continually less; the path of the end of the vector is a series of semicircles whose centres are alternately at O' for the positive swing and O'' for the negative one. For each semi-circle, the angular velocity of its own radius is constant and equal to that of an undisturbed vibration; consequently the phase velocity of OX is not uniform.

This vibration stops suddenly the first time the full extent of the swing is equal to or less than y, for then the friction is able to hold the pendulum at rest against the restoring force.

Since the reversal of the force is equivalent to the application of a force $2F$ in the opposite direction, the energy removed at each reversal, that is during each swing, is:—

$$W_f = 2FX \quad . \quad (36.01)$$

37. *Circular Deviation and Circular Error.*

We have already seen that with a pendulum displaced through an angle θ, the restoring torque is $k \sin \theta$ instead of $k\theta$. Since the sine is always smaller than the angle, the restoring torque is thus smaller than one strictly proportional to the displacement, and the deficiency increases as the angle is made greater. Consequently the rate must go down as the amplitude goes up, giving rise to "circular" deviation, and if the running amplitude should change from any cause we have a circular error of the rate.

Since

$$\sin \theta = \theta - \theta^3/6 + \theta^5/120 -, \text{ etc.} \quad (37.01)$$

and the angles employed with pendulums are small enough to permit us to neglect the last and higher terms, we may write:—

$$F = k \sin \theta = k\theta - k\theta^3/6 +, \text{ etc.} \quad (37.02)$$
$$= k\theta\,(1 - \theta^2/6 +, \text{ etc.}) \quad . \quad (37.03)$$

These two equations express two different ways of dealing with the matter. We can regard the vibration as a true harmonic one, whose restoring force (or torque) is $k\theta$, upon which is superimposed the disturbing force $k\theta^3/6$. Or we may regard it as a harmonic vibration at each instant, but that its controlling "constant" varies from instant to instant according to the value of the expression inside the brackets in the last equation, the fractional deficiency below the value at zero displacement being $\theta^2/6$.

(*To be continued.*)

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Before continuing my remarks upon drills allow me to correct a slip I made in copying out the rough notes for my last article. In the paragraph on P.218 beginning "Too long, etc.," the angle made by the two cutting edges should be 90° to 100° and not 45° to 50° as I said. When the head of a drill is too long and acute it tends to break through too suddenly and stick fast and so runs the risk to be snapped off short.

Last month I made the remark that the ordinary drill used by watchmakers with a bow is not a true cutter but a scraper, and could not be otherwise since it was used in both directions; this, I think, requires some explanation.

In the last resort all true cutters whether planing, boring or turning, are in essence wedges, and as such act by splitting off the material upon which they are operating. In planing, the angle which the front or cutting surface makes with the direction in which the tool is being forced depends upon the nature of the material being operated upon. As a general rule the softer the material the more acute the angle may be; in other words, the more rake or set back from the right angle you can give the tool, the less force it requires to split away the shavings. Compare now the different angles of the carpenter's plane and the cutter of a planing machine shaving off metal. In the case of the latter the acting face of the cutter is almost at right angles with the direction of the force, body of such a person with a compass by moving it up and down the arm and round the wrist; if there is even a trace of magnetism in the body it will deflect the needle either by attraction or repulsion. Another test would be to sprinkle iron filings on some smooth paper and let the supposed human dynamo move his hands about under the paper; the filings would undoubtedly reveal the presence of magnetism by following the movement of the hand. I need hardly say that steel things like knives, keys, rings, etc., should be laid aside, likewise also the offending watch.

If the owner is still unconvinced why not let him carry a watch that is really non-magnetic, that is to say, one with no steel or nickel in the escapement or balance and having a palladium balance spring, not a ferro-nickel spring nor one of any other steel alloy, very few or the watches so lightly termed non-magnetic are entirely immune; they are nearly all liable to become magnetised, temporarily at any rate.

So far as I know, though of course I may be mistaken, these super-susceptible watches are wristlet watches; one does not hear of pocket watches being similarly affected, then get the wearer to carry it in his pocket for a time and observe the effect.

My own opinion, for what it is worth, is that the source of the trouble may be looked for in the unconscious nervous or rhythmic movements of the hand and arm of the wearer; everyone knows how easy it is to modify the motion of the balance by certain movements of the watch, it might be that when worn on the right instead of the left wrist a different result would be obtained.

As to the whistle heard from the speaker of a sensitive radio set on the approach of certain persons, I have heard of the phenomenon, but not being a radio technical expert I do not feel qualified to discuss the question. Doubtless, some of my readers have a much better acquaintance with the subject and could say what it is that takes place within the set on these occasions.

(To be continued.)

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

(Continued from page 274).

The effect on the rate may be calculated by either method; the former one is somewhat simpler and follows on the work we have just done, but the latter permits an approximate elementary proof which we shall give first.

The fractional diminution of control varies from zero at the centre to $\Theta^2/6$ at the end of the swing; the value at the mid-time point is the same as the mean of these two values, namely, $\Theta^2/12$. Now, a constant decrease of control would give a negative rate deviation whose amount is half the fractional reduction of control.

But, in much the same way as the application of a force does so, the mere *act* of decreasing the control on the outward swing, and also of again increasing it on the inward swing, causes a loss of phase beyond that due to the fact that the phase velocity is afterwards less. Consequently, the circular deviation must be greater than $\Theta^2/24$, as estimated from the mean reduction of control, but on the other hand it must be less that $\Theta^2/12$ as given by the maximum reduction of control. The correct value cannot, therefore, be very different from the mean of these two, namely, $\Theta^2/16$. As we shall see, that is its actual value.

From the other point of view, $k\theta^3/6$, together with the smaller terms which we shall neglect, is a disturbing force which is superimposed upon what would otherwise be a true harmonic motion. Leaving out the higher terms, we may write for this disturbing force:—

$$F_d = k\theta^3/6 = k\,(\Theta \sin\phi)^3 \div 6 \qquad (37.04)$$

Since it is the *change* of the force which affects the rate, we must differentiate thus:

$$dF_d = \tfrac{1}{2}k\Theta^3 \sin^2\phi \cos\phi\, d\phi \qquad (37.05)$$

Hence, from equation (35.08), the corresponding portion of the loss of phase is:—

$$d\lambda = (dF_d/k\Theta)\cos\phi \quad . \quad . \qquad (37.06)$$

$$= \tfrac{1}{2}\Theta^2 \sin^2\phi \cos^2\phi\, d\phi \quad . \qquad (37.07)$$

$$= (1/8)\,\Theta^2 \sin^2 2\phi\, d\phi \quad . \qquad (37.08)$$

$$= (1/16)\Theta^2\,(1 - \cos 4\phi)\, d\phi \quad . \qquad (37.09)$$

Hence,

$$\lambda = \int_0^{2\pi} d\lambda = (1/16)\,\Theta^2 \left[\phi - \tfrac{1}{4}\sin 4\phi\right]_0^{2\pi} \qquad (37.10)$$

$$= (2\pi/16)\,\Theta^2 \quad . \quad . \quad . \qquad (37.11)$$

But the deviation is:—

$$\Delta_c = -\lambda/2\pi = -\Theta^2/16 \quad . \qquad (37.12)$$

$$= -52.89 \times 10^{-6} \times (\Theta/100 \text{ arc-minutes})^2 \qquad (37.13)$$

$$= -4.569 \text{ s.d.} \times (\Theta/100 \text{ arc-minutes})^2 \qquad (37.14)$$

The error due to an accidental change, $d\Theta$, in the amplitude is:—

$$\delta_c = d\Delta_c = -(1/8)\,\Theta d\Theta = -(1/8)\,\Theta^2\,(d\Theta/\Theta) \qquad (37.15)$$

$$= -1.058 \times 10^{-6} \times (\Theta/100 \text{ arc-minutes}) \times (d\Theta/1 \text{ arc-minute}) \qquad (37.16)$$

$$= -0.0914 \text{ s.d.} \times (\Theta/100 \text{ arc-minutes}) \times (d\Theta/1 \text{ arc-minute}) \qquad (37.17)$$

$$= -105.8 \times 10^{-6} \times (\Theta/100 \text{ arc-minutes})^2 \times (d\Theta/\Theta) \qquad (37.18)$$

$$= -9.14 \text{ s.d.} \times (\Theta/100 \text{ arc-minutes})^2 \times (d\Theta/\Theta) \qquad (37.19)$$

Fig. 38 shows the amount of the circular deviation for different amplitudes, and also the circular errors due to one arc-minute and one per cent. change of amplitude.

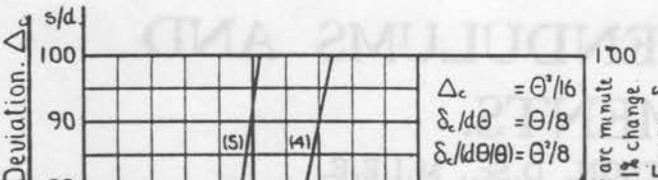

FIG. 38.—*Circular Deviation and Circular Error.*

(1) Circular deviation.
(2) Circular error for one arc-minute change of amplitude.
(3) Circular error for 1 per cent. change of amplitude.
(4) Same as (1), to 10 times larger scale.
(5) Same as (3), to 10 times larger scale.

In the absence of any exact theory as to the influence of the suspension spring upon the circular deviation and error, it is the custom to assume that the above formulæ can be applied to the ordinary pendulum; strictly speaking, they should only be used when the path is a circular arc.

The suspension spring certainly does modify the path to some extent, and it is supposed to do so in such a manner as to reduce the circular deviation. But such experimental evidence as is available seems to show that any such correcting influence is too small to be separated from the errors of experiment.

The position of the balance wheel and of the torsional pendulum with regard to circular error has already been discussed in section 33.

38. *Compensation of Circular Error.*

We shall see later that the circular error caused by an accidental variation of escapement action is usually considerably greater than the direct errors due to the same cause, and that circular error will consequently set the ultimate limit to the accuracy possible with pendulum clocks unless some satisfactory device can be obtained for eliminating it.

The importance of circular error has been recognised from the very birth of the pendulum clock, Huygens himself having invented a corrector for it.

Huygens' pendulum was suspended by two cords, equivalent to a suspension spring of very great flexibility. Cheeks of cycloidal form were fixed on each side of the cords, so as to constrain them, when the pendulum is displaced, to take the curve of the cheek instead of lying along a straight line from the top to the bottom. Huygens showed that this would make the path of the c.g. take the desired cycloidal form, and also that with the cycloidal path the period would not vary with the amplitude.

But the difference between the circular path and the ideal cycloidal one is invisibly small with the amplitudes customary for clock pendulums. With the simple pendulum, the lift with a displacement x along the arc is $L(1 - \cos\theta) = \frac{1}{2}L\theta^2\,(1 - \theta^2/12 +$, etc.$) = \frac{1}{2}\,(x^2/L)\,(1 - \theta^2/12 +$, etc.$)$. With the same displacement, x, the cycloidal arc gives a lift of $\frac{1}{2}x^2/L$ when the cycloid agrees with the circle at the lowest point. The former is less than the latter by the fraction $\theta^2/12$, which is $1/14\,180$ for 100 arc-minutes.

Now, with this displacement, the lift of a seconds pendulum is 0.420 mm., and the difference between the two arcs will only be 0.420 mm. $\div$ 14 180 or 30×10^{-6} mm., but in spite of its smallness it alters the rate by 4.57 seconds per day.

Obviously, a small error in shaping the cycloidal cheeks, or in fixing them in position, may very easily make much greater differences in the level of the c.g. of the bob, and so put Huygens' compensator quite out of action, and moreover, the friction of the cords on the cheeks may cause greater irregularities of rate than those which the device is intended to remove. As a result, the cycloidal cheeks have not survived the test of practical use.

Very many people have tried to eliminate circular error by some special shaping of the suspension spring, such as tapering its width or its thickness, or both, presumably with the idea of getting a better approach to the cycloidal path. But the theories which guided them were extremely vague, and they mostly forgot the escapement errors consequent upon a change of amplitude. None of these have been successful, and there does not seem to be any good reason for any expectation of their success.

About 1850, or slightly earlier, E. T. Loseby* tried another way with a certain amount of success. He fixed a spring in such a position that it pushed the pendulum towards the centre when the displacement to one side exceeded a definite amount. This spring increases the rate of the pendulum to an extent which gets greater as the amplitude is increased, because the spring is then in action during a greater fraction of the time. By a suitable choice of spring, and a suitable adjustment of its position by trial, he succeeded in getting a very small difference of rate when the driving weight was increased from 8 lbs. to 32 lbs., the amplitude being thereby raised from 82 arc-minutes to 162.

He thus balanced one error by another, but the two errors do not vary with the amplitude according to the same law; having got a balance for two different conditions, as above, it would not remain right for intermediate conditions, nor if the amplitude were varied in some other way, such as a change of friction.

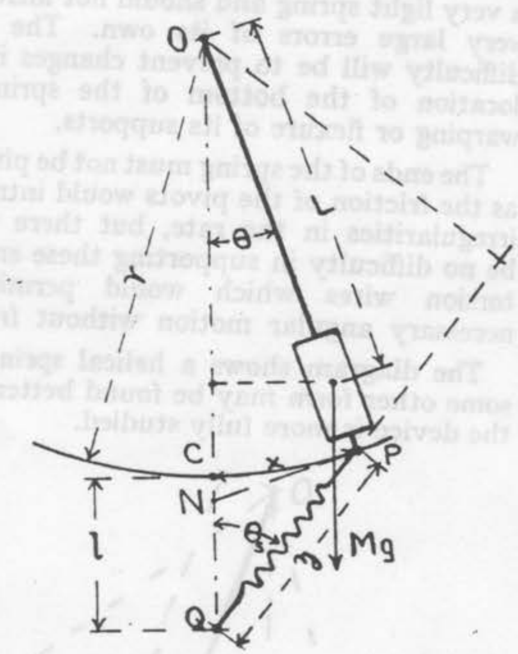

FIG. 39.—*Compensator for Circular Deviation.*

Fig. 39 shows a form of compensator which has the right law, and which has not yet been tried so far as the author has been able to ascertain. The bottom of the bob is tied to the frame by an extensible spring arranged to be in line with the pendulum at zero, and to exert no pull in that position of the pendulum.

As shown below, such a spring gives a positive torque towards the centre whose variation with the displacement follows very closely the same law as the deficiency in the controlling torque which causes the circular deviation. The circular deviation and the circular error can therefore be eliminated if a spring of the correct stiffness be employed.

(*To be continued*).

* See "An Account of the Improvements in Chronometers, Watches and Clocks, by E. T. Loseby." By J. J. Farmer, Coventry. (Printed by Iliffe & Sons, Ltd., Coventry, n.d., but after 1909). The author is indebted to Mr. F. Hope-Jones for drawing his attention to this device and for the loan of the book.

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

(Continued from page 295, vol. lxxi.)

38. *Compensation of Circular Error (continued).*

The correct spring gives a pull of the order of 5 millionths of the weight of the pendulum when stretched 1 mm. It is thus a very light spring and should not introduce very large errors of its own. The chief difficulty will be to prevent changes in the location of the bottom of the spring by warping or flexure of its supports.

The ends of the spring must not be pivoted, as the friction of the pivots would introduce irregularities in the rate, but there would be no difficulty in supporting these ends by torsion wires which would permit the necessary angular motion without friction.

The diagram shows a helical spring, but some other form may be found better when the device is more fully studied.

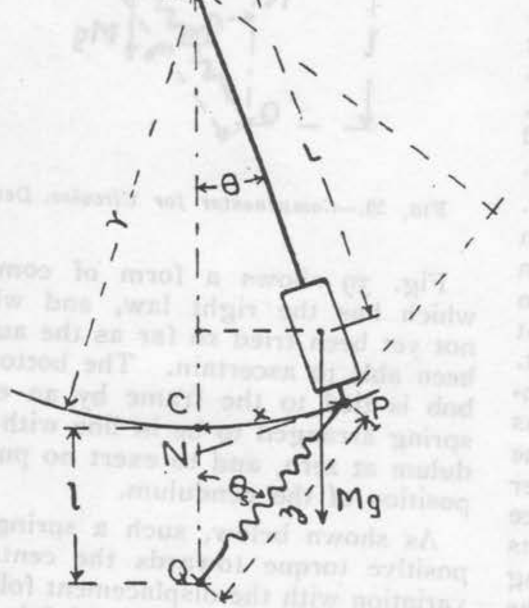

FIG. 39.—*Compensator for Circular Deviation.*

There would be errors in the adjustment of the spring to the correct stiffness and in getting its zero set exactly right, and it is certain that the new attachment will introduce errors of its own. But it seems not unreasonable to expect that the resultant error will be very appreciably less than the uncompensated circular error.

The following is the theory of this compensator.

Let the point P on the bottom of the pendulum be anchored to the point Q on the frame, which is vertically under P when the pendulum is at zero, by means of a spring of such a length that it exerts no pull on the pendulum in the zero position.

Let the radius to P be r, and the pendulum make an angle θ with the vertical in the position shown. If the original and stretched lengths of the spring be l and z, then from the triangle OPQ we have :—

$$z^2=r^2+(r+l)^2-2r(r+l)\cos\theta \quad (38.01)$$
$$=l^2+2r(r+l)(1-\cos\theta) \quad (38.02)$$
$$\doteqdot l^2+2r(r+l)(\tfrac{1}{2}\theta^2-\theta^4/24) \quad (38.03)$$
$$(z-l)=(z^2-l^2)\div(z+l) \quad (38.04)$$
$$\doteqdot r\{(r+l)/(z+l)\}\theta^2(1-\theta^2/12) \quad (38.05)$$

The torque exerted by the compensator is :

$$\tau_s=k_s(z-l)(r+l)\sin\theta_s \quad (38.06)$$
$$=k_s(z-l)(r+l)(r/z)\sin\theta \quad (38.07)$$
$$=k_s(r+l)^2\{r/(z+l)\}(r/z)\times \theta^2(1-\theta^2/12)\sin\theta \quad (38.08)$$
$$\doteqdot k_s(r+l)^2\{r/(z+l)\}(r/z)\times \theta^2(1-\theta^2/12)(\theta-\theta^3/6) \quad (38.09)$$
$$\doteqdot k_s(r+l)^2\{r/(z+l)\}(r/z)\times \theta^3(1-\tfrac{1}{4}\theta^2) \quad (38.10)$$

The gravitational torque is :—

$$\tau_g=MgL\sin\theta=MgL\theta(1-\theta^2/6+\theta^4/120) \quad (38.11)$$

and the discrepancy torque,

$$\tau_d=-MgL(\theta^3/6)(1-\theta^2/20) \quad (38.12)$$

Neglecting higher powers than θ^3, the compensator torque will balance the discrepancy torque if,

$$MgL\theta^3/6=k_s(r+l)^2\{r/(z+l)\}(r/z)\theta^3 \quad (38.13)$$

Or,

$$k_s = (MgL/6) \div [(r+l)^2 \{ r/(z+l) \} (r/z)] \quad (38.14)$$

$$\doteqdot (MgL/3) \div [(r+l)^2 (r/l)^2] \quad . \quad (38.15)$$

taking $z \doteqdot l$.

L may be taken as 1,000 mm., *r* as 1,200 mm., and *l* as 200 mm., in which case k_s is $4{\cdot}7 \times 10^{-6} \times Mg$ per mm. The pendulum G1 has a total mass of 5,100 grams; it would therefore require a compensating spring whose constant is about 24 milligrams weight per mm., or which would stretch 42 mm. with a weight of one gram.

Since the above was written, the author has found that a similar device is employed in the Bulle clocks. The point P is above the bob, and the spring pulls upwards instead of downwards, but the same theory applies if (r − l) be substituted for (r + l).

VIII. Essentials of Escapement Action.

39. *Essentials of Escapement Action.*

The horologist's ideal is a pendulum running absolutely free from disturbing forces of any kind; because of the impossibility of eliminating friction, that idea can never be attained, and we must be content with the nearest practical approach to it.

If the amplitude of the vibration is to remain steady, the pendulum must receive energy from some outside agent at the same average rate as it is wasted by friction. Moreover, the pendulum must also give some sort of timing signal at regular intervals by which the flow of that energy is controlled and the oscillations counted.

There are thus two essential elements in the action of any kind of escapement, namely:—

(*a*) The supply of energy to the pendulum.
(*b*) A timing signal to time the impulses and to operate the counting mechanism.

Both the frictional and driving forces cause deviations of the rate, as will also any additional forces applied by the timing device. Irregularities in the amount of any of these forces or in their timing will cause rate errors.

With the pendulum in a sealed tank kept at reasonably constant temperature, air friction is not subject to change, but solid friction is very erratic and may easily rise to over twice its minimum value. If oil be used, it thickens; if the surfaces be run dry they roughen.

For an accurate clock, it is therefore absolutely essential that the solid friction be kept down to a very small fraction of the total; if the time-keeping is to compete with that of the best existing clocks, that fraction must be well under 1 per cent.

This fact must be borne in mind when deciding upon the choice of the gas in the tank and upon its pressure. A reduction of the total resistance by employing a low air pressure, or having an atmosphere of hydrogen, will make the performance worse, not better, if the result be to raise the proportion of the solid friction above a definite limit. This is often forgotten.

When a clock is driven through gearing, changes in the friction of the train of wheels may cause large variations in the effective driving force, and thus bring about comparatively large errors in the rate. Purely mechanical clocks which aim at a fair degree of accuracy must therefore employ some sort of gravity lever, or its equivalent, to prevent the variations in the driving train from being passed on to the escapement.

40. *Methods of Supplying the Energy.*

The energy required to make up for that lost by friction can be supplied to the pendulum in a number of different ways, such as the following:—

(*a*) Mechanically through sliding pallets.
(*b*) Mechanically through roller pallets.
(*c*) Mechanically through link action.
(*d*) Mechanically through striking pallets.
(*e*) Elastically through a spring.
(*f*) By shifting the suspension point to and fro.
(*g*) By direct electromagnetic action.

Other methods are possible, such as by puffs of air or other fluid, by electrostatic forces, or by radiometer action, but none of these give any promise of being really useful.

Most escapements repeat their cycle every swing, a few every second swing, and others every thirty, or other number of swings. So far as the pendulum is concerned, it is immaterial how frequent the impulses are, for the same energy has to be given over a stated time whether it be given second by second, or in a smaller number of larger chunks.

The rate errors remain exactly the same if we assume, as is reasonable, that the mount of the probable erratic changes in the driving force is proportional to that force.

For the subdivision of time, it is desirable to have an impulse every beat, so that a number of comparisons between different clocks can be made in a short time. But on the other hand a particular escapement may be capable of doling out the energy more accurately in large amounts than in small amounts, or vice versa; or again, stray effects due to wear, shake, springiness, etc., might be less in proportion in one case than the other.

In section 34 we saw that energy was only given to the vibration when the driving force is *changed*. It is therefore convenient to classify escapements according to the nature of the change of the driving force during one swing.

Some types of escapement give intermittent action with sudden changes of force; others give a continuous change during a part, or the whole, of the swing.

Those which change the force at only one point of phase in each swing are thus single-phase escapements. Examples of these will be given later. Others, such as the Graham and the cylinder escapements, apply and remove a force during each swing; these are two-phase escapements.

Those with a greater number of sudden changes, or with continuous change, may be grouped together as polyphase escapements.

41. *Sliding Pallets.*

In most mechanical clocks, the escapement operates through sliding pallets, of which one element is connected, either directly

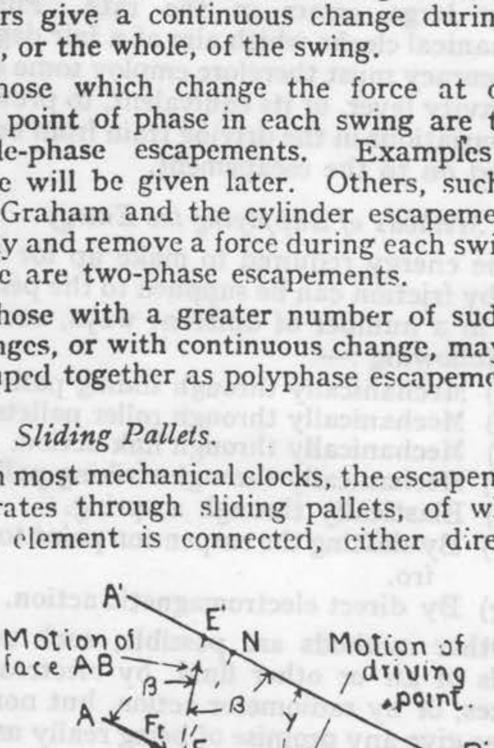

FIG. 40.—*Action of Sliding Pallets.*

or through gearing, to the driving weight or spring, and the other is attached to the pendulum or balance wheel, or connected to it by means of a crutch or lever.

In Fig. 40, AB is the driven element and CD the driving one; at the instant to which the diagram applies, the point E on the former is in contact with the point D on the latter.

We shall suppose that AB is a plane surface, and that it moves parallel to itself in the direction EE^1, while the path of the driver is DD^1. Thus, when AB moves to A^1B^1, E goes to E^1 and D to D^1. EN is the normal to AB through E.

Let AB make an angle β with its own motion, and γ with that of the driver, and the coefficient of friction be μ.

F, the total force exerted by the driver, is made up of the useful component F_n, normal to the surface, and the tangential component $F_t = \mu F_n$, which is required to balance the friction.

During the movement from AB to A^1B^1, the useful work done by the driver is :—

$$W_u = F_n \times EN = F_n \times EE^1 \sin\beta \quad . \quad (41.1)$$

and that absorbed by friction is :—

$$W_t = F_t \times E^1D^1$$
$$= \mu F_n \times EE^1 \sin(\beta+\gamma) / \sin\gamma \quad (41.2)$$

Hence,

$$W_t / W_u = \mu \sin(\beta+\gamma) / \sin\beta \sin\gamma \quad (41.3)$$
$$= \mu(\cot\beta + \cot\gamma) \quad . \quad . \quad (41.4)$$

The efficiency of transmission is :—

$$\eta = W_u / (W_u + W_t)$$
$$= 1 \div \{1 + \mu(\cot\beta + \cot\gamma)\} \quad (41.5)$$

It is obvious that the more nearly the angles β and γ approach 90°, the smaller will be the proportion of the work done against pallet friction. This amounts to the same thing as saying that there should be no more sliding than is absolutely needful.

When the angles become 90°, the two motions are in line and there is no sliding. The pallets have now become striking pallets. As many mechanical escapements require to be timed by one pallet slipping over an edge on the other, it is obvious that some sliding is essential to the action of such escapements, and that striking pallets could not be used with this particular type.

Striking pallets can, however, be employed with electrical methods of controlling the impulse, and also with mechanical clocks employing some form of latch release.

We may assume that the minimum coefficient of friction is 0·1; if then $\beta=60°$, and $\gamma=30°$, which are typical values, cot $\beta=0·577$, cot $\gamma=1·732$, and their sum is 2·31. Thus, in this case, the energy absorbed by friction during the impulse is 23 per cent. of the useful work done by the driver, and $\eta=81\%$. In addition, there is generally a considerable amount of friction loss at the pallets during the dead, or locking, period, and in the pivots of the escapement.

As a rule, the total solid friction loss of such escapements exceeds the net energy given to the pendulum, and consequently they are quite unfitted for accurate timekeeping.

The geometry of any particular escapement mechanism will involve a particular angle between the direction of the two motions, E^1ED in the diagram. Since this angle is equal to $180° - (\beta+\gamma)$, it follows that $(\beta+\gamma)$ is fixed for that particular design, and consequently any increase in β requires an equal decrease in γ.

To find the condition for least friction loss during the impulse, we must equate to zero the differential coefficient of equation 41·4, thus :—

$$O = \operatorname{cosec}^2 \beta d\beta + \operatorname{cosec}^2 \gamma d\gamma \quad . \quad . \quad (41·6).$$

Or, since $d\beta = -d\gamma$, $\operatorname{cosec} \beta = \operatorname{cosec} \gamma$ (41.7)

Hence $\beta=\gamma$ . . . . (41.8)

We thus see that the inclination of the impulse plane should be such that it makes equal angles with the direction of its own motion and that of the driving point, but a moderate departure from this condition makes very little difference.

With escapements having an anchor, it is usual to have the two motions at right angles, which makes $(\beta+\gamma)=90°$. This would give 45° as the best value for these two angles, giving 2·00 for the sum of the cotangents, as compared with 2·31 obtained above for the customary angles. The use of 60° and 30°, instead of 45°, thus causes an increase of the friction loss during the impulse of 15 per cent. of its own amount. As the friction loss during the impulse is only a portion of the total caused by sliding, the above percentage would be two or three times smaller if reckoned on the total solid friction. Consequently it is of no great importance.

If the pallet drives the wheel, as happens during recoil, the same equations apply, but W_u is then the total work done by the pallet, and the energy received by the wheel is $W_u - W_f$.

The efficiency is now :—

$$\eta = (W_u - W_f) / W_u$$
$$= 1 - \mu (\cot \beta + \cot \gamma) \quad . \quad (41.9)$$

(*To be continued.*)

Mill Hill Observatory. A new observatory, erected by the London University at Mill Hill, was opened on the 8th ult. by Sir Frank Dyson, the Astronomer-Royal.

Some time ago a 24in. reflecting telescope was placed at the disposal of the University by Mr. J. G. Wilson. It had been constructed for his father, Mr. W. E. Wilson, F.R.S., at his home in Ireland. It was the generosity of Mr. Wilson that made the observatory possible.

The new observatory is completely up to date. Professor G. N. L. Filon, the director, and the observer, Mr. C. C. L. Gregory, have acted as advisers to the architect throughout.

The Wilson telescope has been adapted so that its clockwork can be re-wound by electricity, and the dome will also be rotated by electricity.

On certain days the public is to be admitted to the observatory, in parties of not more than 12.

Glastonbury's Clock. The ancient striking clock of Glastonbury, in St. John's Church tower, has recently broken down completely, after over 300 years of service.

The clock is unusual, being without a face and hands externally, owing to the construction of the handsome tower not permitting of the customary features of a public clock.

For some years the works of the time and striking apparatus, a ting-tang arrangement, have been kept going with wire and string ties.

Repair proving to be impossible, the churchwardens issued a public appeal for subscriptions to provide new works.

The appeal has now been withdrawn, Miss Rocke, the Lady of the Manor and Hundred of Glaston Twelve Hides, having generously come forward and offered to provide a new apparatus with Westminster chimes, at her ow noost.

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

(*continued from page* 54.)

42. *Pallet and Roller.*

By replacing the sliding plane by a roller, the friction is shifted to the pivot of the roller, where the sliding is much less and the friction loss is correspondingly reduced.

Let us suppose that the driving point is constrained to move in a vertical line, and that the component, in that direction, of the force it exerts is constant. The constant vertical force is commonly derived from a weighted lever. Further, let the pivot of the roller move in a horizontal line, and let the effects of the inertia and weight of the wheel be negligible.

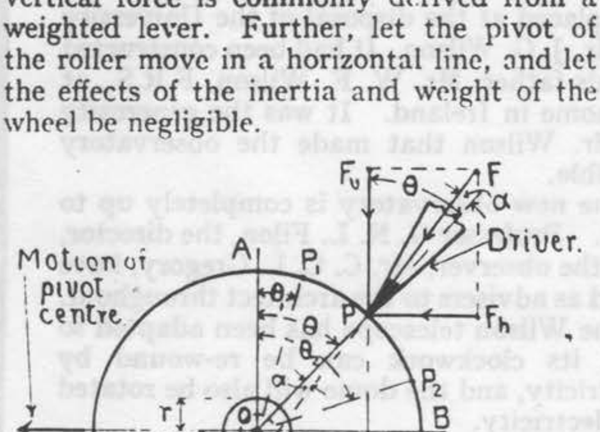

FIG. 41.—*Action of Pallet and Roller.*

In Fig. 41, the driving point is at *P*, where the radius *OP* makes an angle θ with the vertical. If there were no friction, the force *F* would act along the radius. But the friction resists the rotation of the roller with the torque :—

$$\tau = \mu F \times r = F \times (\mu r) \quad . \quad (42.1)$$

where r is the radius of the pivot and μ the effective coefficient of friction for the bearing.

Since the driving force must balance this torque, it will not pass through the centre of the roller, but at a distance μr below it. Thus the direction of *F* will touch the dotted circle whose radius is μr, and it will make an angle α with the radius *OP*, where :—

$$\alpha = \tan^{-1} \mu r / R \quad . \quad . \quad . \quad (42.2)$$

Since *R*, the radius of the roller, is made much greater than r, the angle α is always a very small one.

The vertical and horizontal components of *F*, are F_v and F_h ; the former is supposed to be constant, and the latter must therefore vary with the position of *P* so as to keep *F* tangetial to the dotted circle. From the diagram we see that :—

$$F = F_v \sec(\theta - \alpha) \quad . \quad . \quad (42.3)$$

If the action begin at P_1 and end at P_2, whose angular positions are θ_1 and θ_2, the total work done is :—

$$W_t = F_v R (\cos\theta_1 - \cos\theta_2) \quad . \quad (42.4)$$

and the energy absorbed by friction is :—

$$W_f = \int_{\theta_1}^{\theta_2} \tau_f d\theta = \mu r F \int^{\theta_2} \sec(\theta - \alpha)\, d\theta \quad (42.5)$$

$$= \mu r F_v \log_\epsilon \left[\tan\left(45° + \tfrac{1}{2}\theta_2 - \tfrac{1}{2}\alpha\right) \cot\left(45° + \tfrac{1}{2}\theta_1 - \tfrac{1}{2}\alpha\right) \right] \quad . \quad . \quad (42.6)$$

Hence,

$$W_f / W_t = \mu (r/R) \log_\epsilon \left[\text{ditto} \right] \div (\cos\theta_1 - \cos\theta_2) \quad . \quad . \quad . \quad (42.7)$$

In the particular case where the action begins at the top and ceases at the horizontal radius, this becomes :—

$$W_f / W_t = \mu (r/R) \log_\epsilon \left[\cot \tfrac{1}{2}\alpha \cot\left(45° - \tfrac{1}{2}\alpha\right) \right] \quad . \quad . \quad . \quad (42.8)$$

$$= \mu (r/R) \log_\epsilon (2R/\mu r) \quad . \quad (42.9)$$

because α is small enough to permit us to write $\cot(45° - \tfrac{1}{2}\alpha) = \cot 45° = 1$, and $\cot \tfrac{1}{2}\alpha = 2 \cot \alpha = 2R/\mu r$.

Let us take $\mu = 0.1$, as before, and $r/R = 1/50$. Then $2R/\mu r = 1{,}000$, = whose natural logarithm is 6.91. This gives

$$W_f / W_t = 6.91/500 = 1.4\% \quad . \quad (42.10)$$

A pivot, or other means of constraining it, will be required to guide the driver along the desired path. This will add a little more to the friction loss and bring it up to say

1.5%. This is not quite up to the standard set in section 39, but is not far from it.

The pallet and roller have thus very little friction, but sufficient to set a definite limit to the accuracy of time-keeping attainable, and they have a slight handicap in another direction.

Since the force applied to the pendulum acts outwards, its gradual *increase* during the impulse *abstracts* energy from the vibration in spite of the fact that the kinetic energy is being increased. The *removal* of the force when the pallet falls clear *imparts* energy to the vibration, and the amount so given must exceed that required to maintain the vibration by the amount previously taken away. Thus, the gross energy handled by the escapement is much greater than the net energy given to the pendulum.

Now we shall see later that the magnitude of the probable rate errors due to erratic changes in the forces or their timing, is proportional to the gross amount of energy handled, irrespective of sign. On this score, the pallet and roller escapement is not so good as one which always gives energy and never takes any back.

Nevertheless the pallet and roller are employed in the Shortt free pendulums, as constructed by The Synchronome Coy. Ltd., and these are far and away the most accurate time-keepers yet made. They have a ratio of nearly 50 for the radii of the roller and pivot, and so the numerical results given above will apply approximately to them.

The link of the link escapement may be regarded as a roller which never goes out of action and which can transmit pull as well as push. Because of the continuous action, it has a much bigger friction loss (about 10 times) than the roller, and is therefore less suited for primary clocks.

But it lends itself to particularly effective methods of automatic control by the Post Office Time Signals from Greenwich.*

43. *Maintenance by Direct Spring Action.*

Clocks in which the driving force is given by a spring through the agency of pallets do not fall into this class, but into that which includes the type of pallets they employ.

But a few escapements supply the energy through a spring without the intervention of pallets, and so avoid most of the friction associated with them. In such escapements, the energy to be given is first stored as strain energy in the spring and its amount is definitely determined by the strain imparted to the spring.

The simplest escapement of this type is the one due to Mallock which will be described in the next chapter. In it, two light springs connect the ends of a horizontal arm attached to the pendulum to those of a rocking horizontal bar pivotted on the frame and rocked up, or down, by electromagnetic means at a definite point on each swing.

Riefler employs the suspension spring itself as a means of conveying the energy to the pendulum. The chops carrying the suspension spring are not fixed but are carried on a vertical rocking lever resting on knife edges in line with the pivotting point of the pendulum.

As the pendulum swings out from zero, this lever goes with it owing to the reversal of the torque on the spring, which formerly held it against a stop. After a certain amount of movement in this way, an escapement connected to the lever is released and forces the lever quickly back to its normal position, where it is held until the next operation.

In this way, a definite amount of energy is stored in the spring at each operation and serves to keep the pendulum going until the next.

With direct spring action, no friction losses are necessarily involved in the act of transferring the energy, but with Riefler's device there is some friction loss at the knife edges and in the escapement which are essential parts of his arrangement. Nevertheless, his clock is the best purely mechanical clock which has yet been devised and held the record for accurate time-keeping until the advent of the free pendulum.

44. *Maintenance by Movement of Suspension Point.*

It is quite a common experiment to get up a vibration of a simple pendulum by moving the top of the string to and fro, but so far as the author can ascertain, this principle has never been applied to a clock pendulum.

Yet there is little doubt that it is the best of all methods for supplying the energy, for it is inherently devoid of friction so far as the pendulum is concerned. There must, of course, be some friction in the apparatus for moving the suspension point to and fro,

* See paper already cited on page 128, vol. LXXI.

but the energy so absorbed is not taken from the pendulum either directly or indirectly by abstraction from the store which would otherwise be passed on to the pendulum; it simply increases the work to be done by the agent which causes the to and fro motion without affecting the amount of energy which reaches the pendulum.

The only way in which that friction can possibly affect the rate of the pendulum is by altering the time which elapses between the arrival of the timing signal which controls the movement and the completion of that movement.

This elimination of solid friction removes entirely one of the main sources of erratic variations in the rate of the pendulum.

The driving force is derived from the forces controlling the vibration, which in the case of a pendulum depend mainly upon its weight, and to a small extent upon the suspension spring. As we shall see later, the amount of the force depends upon the amount of the movement of the suspension point, and the energy given depends also upon the position of the pendulum at the instant at which the movement occurs.

In the next chapter details are given of a simple escapement working on this principle, which the author has devised and is now constructing, with the aid of a grant from the Colston Research Society. It is believed that with this device the number of possible sources of erratic variation has been reduced to the absolute minimum, and that the two which are left can be so controlled as to give a performance appreciably better than even the wonderful results now being obtained with the Shortt free pendulum.

What is essentially the same principle has been applied to "free balances" for chronometers by Robert Leslie (1793), Gowland (1849), Benoit (1853), Hillgren (1882), and Riefler (1889), but as these are mechanical escapements, some friction is unavoidable in the locking device.*

45. *Direct Electromagnetic Action.*

In this class are included all methods of driving the pendulum electromagnetically without the intervention of pallets. Such have an electromagnet, or solenoid, acting on an iron core, a permanent magnet, another solenoid or electromagnet, one member being attached to the frame and the other to the pendulum.

The electric current is made and broken, or reversed, at the proper instants by contacts controlled by the pendulum.

There are a great many devices of this kind, but it does not seem probable that any such driving mechanism will ever produce a clock of the highest accuracy; the variation of the driving force with the state of the battery is far too great.

To be appreciably better than the best existing clocks, the irregularities of the rate must be kept down to one or two milliseconds per day. Under the most favourable conditions, this would entail a constancy of the driving force within a few parts in ten thousand, and there is no means of keeping the battery constant, day after day, to anything like this degree. A constancy of 5 per cent., more than 100 times rougher, would need continual attention and be costly in maintenance.

Recently Max Schuler* has described a "free" pendulum employing a very neat form of direct electromagnetic drive, and he proposes to get over this difficulty by keeping a continuous photographic record of the amplitude and applying a correction accordingly. For the accuracy mentioned above, he will have to estimate his mean amplitude correct to within one second of arc.

46. *The Impulse-Timing Signal.*

By some means or other, the pendulum must control the timing of the escapement action; this impulse-timing may be done by purely mechanical action, or some electrical device may be employed for the purpose.

In most cases, the escapement action is directly controlled by the pendulum, but in some, such as the Shortt free pendulum, it is done indirectly through the agency of a second pendulum which is itself controlled by signals from the main pendulum.

The following methods are available for getting the impulse-timing signal:—

With mechanical release.

(*a*) Pallet slipping over an edge.
(*b*) Pallet lifting a detent.

* For descriptions and illustrations of these escapements, see pages 144-148 of R. T. Gould's delightful book, *The Marine Chronometer* (Potter, London, 1923).

* "Eines neues Pendel mit unverandlicher Schwingungszeit." (*Zeits. fur Physik.*, vol. 42, part 7, pages 547-554. April, 1927.)

With electromagnetic release.

(*c*) Solid electric contacts.
(*d*) Mercury contacts.
(*e*) Sparking across a gap.
(*f*) E.M.F. induced in a coil.
(*g*) Electrostatic induction.
(*h*) Beam of light and a photo-electric cell.
(*i*) Beam of heat and a thermopile.

With purely mechanical methods, some slipping is an essential feature of their action and there must consequently be more or less solid friction. The objections to this friction have already been discussed in section 39.

In addition, further irregularities are caused by wear of the pallet surfaces, and by end shake when these surfaces are oblique to the spindles. These cause a change in the position of the pendulum at the instant of release, and thereby cause errors in the rate.

The mechanical clock has been developed by skilled craftsmen and students of the theory for well nigh three centuries, and has now reached a stage of perfection at which further important improvement seems unlikely. The best mechanical clocks give an accuracy amply sufficient for ordinary purposes, but are far behind the best electric clocks and are likely to remain so.

47. *Solid Electric Contacts.*

To make a good electric connection with solid contacts, the pressure should not be less than 10 grams weight, and more is to be preferred.* From the electrical point of view, it is also desirable to make the two elements of the contact rub over one another, but where accurate time-keeping is required this would be absolutely inadmissible when the energy so wasted is derived from the pendulum, or from the store which would otherwise be passed on to the pendulum in a definite number of impulses.

For a reliable make and break, the movement whilst the pressure is on could hardly be less than 0.1 mm.; the very least amount of energy which would suffice to make a certain electric connection with solid contacts is thus of the order of 100 ergs, which is more than what the pendulum would require to keep it swinging with an amplitude of 100 arc-minutes.

* But Mr. E. C. Atkinson tells me that he has successfully run a contact for two years with a pressure of about 0.2 gram weight, the current being just over 1 milliampere.

If this energy be taken from the pendulum, and is wasted, that supplied by the escapement must be correspondingly increased, and would thus be several times that used by the pendulum itself. The probable escapement errors would be magnified to the same extent.

But in addition to this, the contact forces would themselves be subject to erratic variations, both of amount and of timing, and would give rise to an entirely fresh series of errors greater than those due to the escapement itself.

Such an arrangement is thus quite unsuitable for a clock where great accuracy is required.

If the energy be returned to the pendulum on the backward stroke, the escapement errors would not be magnified, but the additional errors would remain.

Matters are very much better when the contacts serve also as driving pallets, because then the driving forces are also the contact forces. As no additional forces are needed for the contacts, the additional errors due to contact forces are eliminated. Difficulties arise from the fact that the forces which are sufficient to maintain the vibration are too small for a reliable contact when an impulse is given every second.

(*To be continued.*)

ERRATA.

In equation 41.1 on page 53, the work done whilst the plane moves through the distance E'N parallel to itself was omitted. This and the subsequent equations should read :—

$$W_u = F_n \times EN - F_t \times E^1N = F_n \times EN (1 - \mu \cot \beta) \quad . \quad . \quad (41.1)$$

$$W_t = F_t \times E^1D^1 = F_t \times (E^1N + ND^1) = \mu F_u \times EN (\cot \beta + \cot \gamma) \quad (41.2)$$

$$(W_u + W_t) = F_n \times EN (1 + \mu \cot \gamma) \quad (41.3)$$

$$W_t / W_u = \mu (\cot \beta + \cot \gamma) \div (1 - \mu \cot \beta) \quad . \quad . \quad . \quad . \quad (41.4)$$

$$\eta = W_u / (W_u + W_t) = (1 - \mu \cot \beta)/(1 + \mu \cot \gamma) \quad . \quad . \quad . \quad (41.5)$$

During the recoil, the efficiency is :—

$$\eta^1 = (1 - \mu \cot \gamma)/(1 + \mu \cot \beta). \quad (41.9)$$

The percentages quoted in the top paragraph of page 54 should be corrected to 25% and 80%. Equations 41.6 to 41.8 also require alteration, but the final numerical result is only slightly affected.

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

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(*Continued from page* 119.)

But there are errors due to alterations in the phase of the contact due to wear. Owing to the action of the current, even when that is very small and the contact faces are of platinum, and there is no visible sparking, the surfaces roughen and wear more rapidly than they would if the current were absent.

To reduce the wear of the contacts to a minimum, all magnets should be shunted by non-inductive resistors, and the contacts should have in parallel with them a condensor in series with a damping resistor. The shunts should have a resistance of about four to five times that of the coil they shunt, which would limit the momentary voltage rise across the coil to four or five times the normal.

The condensor may have a capacitance of one or two microfarads, and its damping resistor a resistance about the same as the remainder of the circuit. When the contacts are open, the condensor is charged to the supply voltage, and it is discharged through the contacts when they close. Without a damping resistor, this discharge current may momentarily be very large, and practically all the energy stored in the condensor is spent in heating and burning the contacts. With a damping resistor of the value given, the discharge current is restricted to the normal current carried by the contacts, and the energy is spent in it. At the same time, when the contacts first open, the main current can continue to flow via the condensor without the voltage across the contacts exceeding that of the supply.

That the condensor discharge can do a surprising amount of damage is shown by a recent experience of the author's. An experimental pendulum makes contact every second for its own maintenance, and takes a current of 0.1 ampere from a 40 volt storage battery. The magnets are shunted by a non-inductive resistor of about five times the coil resistance, and the contacts by a condensor of 2 microfarads in series with about 11 ohms. Nevertheless, in the course of 16 days, new platinum contacts were badly damaged, the positive one having grown a pip about ½ mm. high, and the negative one a pit of about the same depth. On raising the damping resistor to 500 ohms. the trouble disappeared, and no appreciable wear occurred in a month.

The effect of a given amount of wear on the impulse-timing depends upon the velocity with which one contact face is moving relatively to the other just before they strike, but even with the best designs a very small amount of wear will result in an appreciable change of rate.

To give an example, with the author's link escapement a wear of only one mil will cause a rate error of about 1 s/d; although the contacts are of platinum, the current through them only 60 mA, and the circuit is provided with discharge resistors for the magnets and a condensor in series with a damping resistor across the contacts, these arrangements being so effective that even in the dark there is very little visible sparking, yet erratic changes of rate of several tenths of a second per day occur from time to time because of changes in the contact surfaces. This fact, and the friction in the pivots, constitute the defects of this particular escapement.

Most electric pendulums combine the contact and driving forces. There are very many such devices, beginning three quarters of a century ago, but only a few names need be mentioned:—Lias (1854), Tiede and Knoblich (1867), Gill and Cottingham (1880), Hope-Jones (1895).

Hope-Jones impulsed the pendulum every half-minute only, and to him must be given the credit of making the electric clock a sound commercial article instead of a laboratory instrument.

In the Shortt free pendulum (1921) the arrangement is slightly different. The release of the gravity lever which maintains the motion is timed by a contact on a second pendulum, which is itself controlled by the main pendulum through an electric contact which is actuated by the gravity lever after it has fallen clear of the roller on the pendulum.

When released by the slave pendulum, the lever pallet falls on the top of the roller when the main pendulum is at the centre of its swing, and the subsequent changes in the driving force are controlled by the motion of the roller away from the pallet, being timed by the changing inclination of the radius to the point of contact.

Since the driving force with the pallet near the top of the roller is very small, and in any case the application of a force when the pendulum is close to its central position produces but little deviation of the rate, the effects of slight variations of phase difference between the two pendulums are not appreciable.

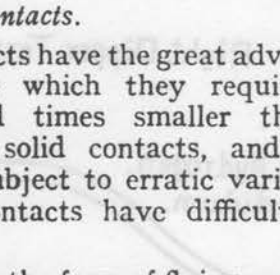
48. *Mercury Contacts.*

Mercury contacts have the great advantage that the forces which they require are several hundred times smaller than is necessary with solid contacts, and these forces are less subject to erratic variations. But mercury contacts have difficulties of their own.

They may take the form of flying contacts, dipping contacts, or tilting contacts.

The flying contact (see Fig. 42) consists of a globule of mercury projecting above the rim of the containing vessel so that a needle

Fig. 42 Flying Mercury Contact.



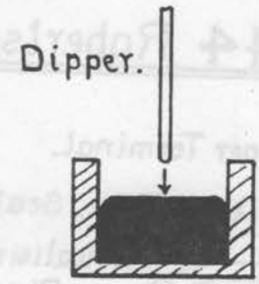
point moving horizontally can touch the mercury without fouling the container. Such contacts cannot permanently give a very high degree of accuracy of timing, because the surface struck by the point makes but a small angle with the direction of motion.

A small change in the relative levels of the mercury and point, which might be caused by loss or oxidation of the mercury, by wear of the point, or by change in the frame which locates the container, would cause a much larger change in the position of the pendulum when contact is established.

This is discounted to some extent by the fact that such contacts can be placed at the bottom of the pendulum where the velocity is a maximum, which is not possible with other forms of mercury contact.

But the flying contact is liable to throw off small droplets, with the result that the metal gets into all the places where it is

Fig. 43 Dipping Mercury Contact



most objectionable, and the level untimately falls so low that the contact fails. It is quite possible to arrange a simple pump, operated electrically one stroke at each impulse to the pendulum, so as to refill the container at each stroke, but this complicates matters.

A modification of the flying contact consists of a horizontal jet of mercury directed across the path of the needle. The contact surface is now perpendicular to the motion of the needle and the accuracy of timing improved. But it seems very doubtful whether the gain would be sufficient to warrant the extra complication and expense, to say nothing about the scattering of the mercury.

The dipping contact (Fig. 43) is much better than the flying one, for the motion of the dipper can be made normal to the mercury surface by having the latter level with the pivotting point of the pendulum, and the walls of the container can be made high enough to prevent any loss of mercury.

But even this arrangement does not give absolute constancy of the position at which electrical connection is made because of dust on the surface of the mercury and oxidation and wear of both mercury and dipper.

(*To be continued.*)

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(*Continued from page 140.*)

Tests show an uncertainty in the position of contact of the order of 0.1 mm. due to the film on the surface of the liquid. In fact, it is possible for the dipper to depress the surface sufficiently to produce a visible dimple without getting electrical connection, and to get electrical connection without any visible signs of contact.

An amalgamated dipper is much superior to a dry one, and it is expected that with a well amalgamated platinum point, protected from oxidation, and kept free from dust, a very high degree of permanence of the mean position of contact can be obtained, although there may be minor variations between individual dips.

With a "dry" point, the surface tension makes the particles of dust collect round the point of the dipper, but with an amalgamated ("wet") surface they are driven away from the dipper.

For a permanent contact, the dippers must be of platinum or iridium, as no metal which can dissolve in the mercury may be employed. Iron and steel satisfy that condition, but they do not amalgamate and give a much more erratic connection than platinum.

Troubles due to dust can only be eliminated if the mercury be sealed up, which is impossible unless both electrodes can be fixed together. This gives a contact which is made by tilting in one direction and broken by tilting in the opposite way.

With such enclosed contacts it is always advisable to fill the tube with hydrogen, or some inert gas, in order to prevent oxidation of the mercury and of the dippers, if such be used.

In the tilting contacts available commercially, which are used for certain time switches and other devices, one electrode is in a little pool of mercury near one end of the tube and the other connects to the main mass of mercury. When the tilt of the tube is sufficient, the two pools of mercury come together and complete the circuit. It is quite fascinating to watch the way in which they immediately coalesce the instant contact begins.

Unfortunately with this type of contact, the amount of reverse tilt needed to separate the mercury again is greater than the angular motion of the pendulum, and so it cannot be applied to our particular purpose. The author has, therefore, constructed one to suit our needs.

A diagram of this contact is given in Fig. 44; two dippers are sealed into glass bulbs connected by two glass tubes, and have a

Fig. 44 Robertson Tilting Mercury Contact.

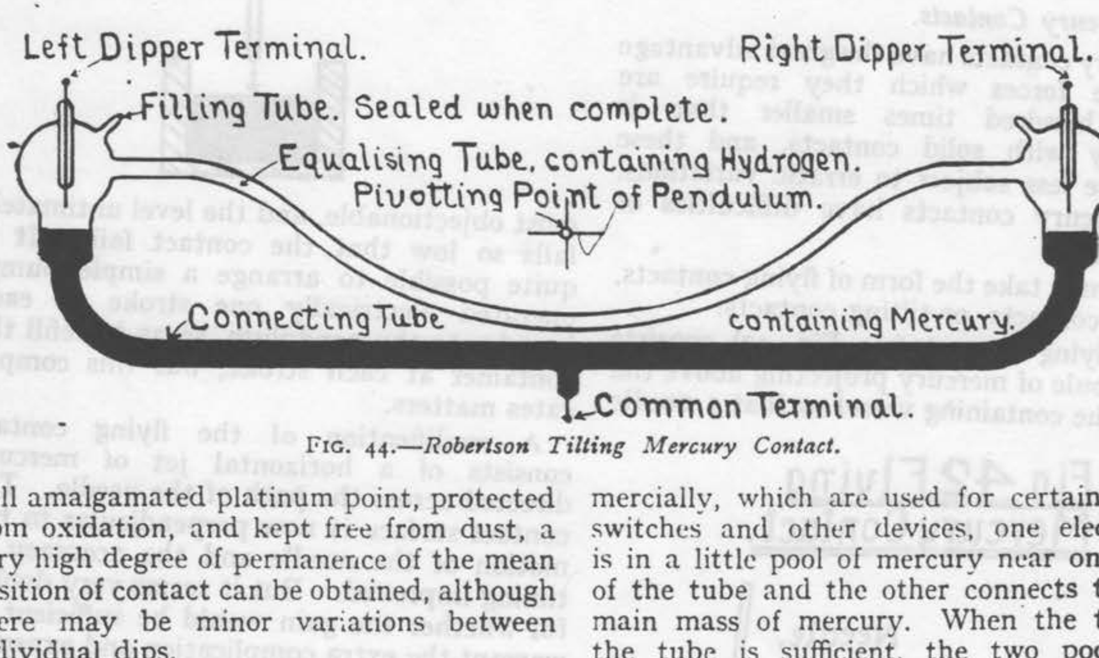

FIG. 44.—*Robertson Tilting Mercury Contact.*

common pool of mercury in the lower tube and bottoms of the bulbs. The whole glass construction is attached to the pendulum between twin rods, with one bulb on each side of the centre.

Approximately, the level of the mercury in the two bulbs remains constant as the pendulum swings to and fro; the dippers thus act very nearly as if the mercury had been supported from the frame instead of from the pendulum.

The contacts are of platinum, the gas hydrogen, and the circuits arranged so that the current is never broken at the pendulum contact; it is therefore believed that a very high degree of permanence will be obtained.

The minor variations from contact to contact which have been already mentioned may possibly cause irregularities of two or three milliseconds between the times given by the individual members of a series of signals, but will not affect the mean nor the rate of the pendulum. When making comparisons of the time given by the clock and some other event, these irregularities can be eliminated by taking sufficient observations.

Suppose that the dippers are placed at a radius of 200 mm., which is really rather more than is quite convenient; then a movement of 1 arc-minute of the pendulum will raise one dipper by 0.06 mm. With an amplitude of 100 arc-minutes and contact made at about 71 arc-minutes from the centre (that is at phase 45°), a change in the relative effective levels of the dipper and mercury of the amount just mentioned (0.6 mm.) would alter the time at which the contact is made by 4.5 ms. The effect of this on the rate will, of course, depend upon the escapement.

49. *Other Electrical Methods.*

With the spark-gap method the gas pressure must be sufficiently low to permit sparking between a needle and a knife edge when these pass sufficiently close together. The needle would be fixed at the foot of the pendulum and the knife-edge at the place where it is required to make connection.

The method would be used in conjunction with a glow relay.* That relay has three electrodes in neon gas. The two outer electrodes are in circuit with a relay of the ordinary type and a supply of about 200 volts, their distance apart is sufficient to prevent any discharge from passing when the intermediate electrode is at, say, the mean potential. The discharge can be started by a sufficient alteration of the potential of the third electrode, and is afterwards extinguished by the operation of the ordinary relay.

As the release current can be kept down to one microampere, and the main current may be as large as 50 milliamperes, the glow relay forms a very attractive type of trigger release. Such a very small release current would not cause much wear of the needle point, but the voltage required to ensure sparking with a reasonable tolerance in the setting of the minimum gap is probably a sufficient objection to prevent the method being tried.

The spark gap method also suffers from a "fringing" effect. As the two elements approach one another the field between them increases more or less gradually until the breakdown point is reached. Because of this gradual increase of field, the exact position at which break-down does take place will vary with the voltage across the gap.

Method (f) contemplates the operation of the release through relays by means of the E.M.F. induced in a coil of wire by a permanent magnet when one moves with the pendulum and the other is fixed to the frame.

Since the energy of the induced current comes from the pendulum, it must be kept down to an absolute minimum for the reasons already discussed; since the permissible energy is too small to operate the most sensitive telegraph relay, and the maximum voltage which can be conveniently generated in the coil is far too small for the glow relay, magnification by means of wireless valves would be required, which would lead to serious expense in upkeep and to frequent stoppages from broken filaments.

In this case also there is a fringe effect, which when combined with variations in the sensibilities of the valve and other relays will cause irregularities in the timing.

The electrostatic method would vary the potential of the intermediate electrode of a glow relay, or of the grid of a wireless valve, by the changes of electrostatic capacity of two plates when one moves with the pendu-

* Richter and Geffcken. "Das Glimmrelais." *Zeits. for Technische Physik.* No. 12, 1926, pages 601-606.

lum relative to the other which is fixed to the frame. This method seems even less practical than the previous two.

50. *Optical Methods.*

Optical methods have been employed by several people, for instance Siegl,* Ferrié and Jouaust,† and Schuler.‖

This method, in which a beam of light is reflected by a mirror on the pendulum into a photoelectric cell, gives the greatest accuracy of timing, combined with the absence of all forces which would disturb the pendulum.

The current obtained from the photoelectric cell when light is admitted to it is very small, (of the order of one microampere) and must be magnified many thousand times before it can be employed for operating the escapement.

The great drawback to the method is the expense of running the lamp and valves continuously and the liability to stoppage through broken filaments. There are, moreover, possibilities of irregularities in the relays which are employed in the final stage of amplification.

It would seem probable that the best way of all is to employ the enclosed mercury contact for the maintenance of the motion, but to use the optical method for getting the signals required for comparison of the clock time with the event to be timed. The running of the clock would then be independent of the optical devices, which would be switched on only when observations are to be made.

The thermal method is similar in principle, but would employ a thermo-junction in place of a photoelectric cell. But even the Moll vacuum type of thermo-junction is far too sluggish to be of use.

A very old attempt to get the impulse-timing signal by means of radiation was made by Gimingham in connection with Gill's escapement and is described by the latter in his Report to the B.A. in 1880.*

Solid contacts of carbon were employed, one member being attached to a light arm carrying a radiometer vane and controlled by a tiny magnet or by a light glass spring which holds the contacts together. The whole thing is enclosed in a glass tube which is exhausted to a Crooke's vacuum.

A strong beam of light is thrown on the vane across the path of a screen carried by the pendulum. When the light falls on the vane, the arm swings round and opens the contact. When the beam is cut off, the contact is remade by the control.

As might be expected from the extreme minuteness of the forces which can be obtained by radiometer action, the contact proved unreliable. The clock as actually made by Cottingham used the driving pallets, which are of the striking type, as electric contacts also, and in this form it gave really good results.

51. *Escapement Errors.*

In sections 34 and 35 we saw that the mere *application* of a force F at an instant when the amplitude is X, the phase φ_o, and the displacement x_o, gives energy to the vibration of the amount :—

$$W_e = -FX \sin\varphi_o = -Fx_o \text{ (Eqn. 34.09)} \quad (51.1)$$

and that it causes a loss of phase

$$\lambda = (F/kX) \cos \varphi_o \quad (\textit{Eqn.}\ 35.03) \quad (51.2)$$

If this force be removed at the corresponding point on the return swing, and the whole operation be repeated every cycle, the rate deviation caused thereby is :—

$$\Delta = -\lambda/\pi = -(1/\pi)(F/kX) \cos \varphi_o. \quad (55.3)$$

Now, we are not concerned with the amount of this deviation, for it is allowed for in the calibration of the pendulum, but any variations in it do concern us for they become rate errors.

With respect to *each* application of force

* K. Siegl, "New Principle for Electric Clocks." (*Zeits. Instrumentenk.* Beib. 9. pp. 81-85. May 1st, 1904.)

† G. Ferrié and R. Jouaust, "The Use of Photoelectric Cells in the observation and maintenance of Astronomical Pendulums." (*Comptes Rendus.* 180 pages 1,145-1,148. April 14th, 1925.) Also Roy. Soc. Edin. Proc., XLV., pages 261-268. June 8th, 1925.

‖ Paper referred to on page 81 of MS.

* David Gill. B.A. Report, 1880, pages 56-61.

(that is to each change of the total force acting) there are thus three sources of erratic variation, namely :—

(*a*) the force itself may vary in amount.

(*b*) the timing of its application may alter.

(*c*) the amplitude may change.

Any variation in the force, or in the phase of its application, will affect the deviation produced by that force because of the change in the corresponding factor of λ. This is the Primary Direct Error.

But, in addition, the original variation will modify the amplitude because it will change the energy supplied by that force. The change of amplitude further modifies the deviation due to the force we are considering because X is a factor of λ, (the Amplitude Error) and also because it affects the phase corresponding to the position at which the impulse-timing signal is given. (The Timing Error.) These two are the Secondary Direct Errors.

But in addition to these changes in the deviation caused by the force concerned in the irregularity, the change of amplitude also produces changes in the deviations due to every other disturbing force ; such forces may be other escapement forces, frictional forces, or the discrepancy forces which produce circular deviation. These are the Indirect Errors, and their amount depends upon the other disturbing forces.

Since the amplitude cannot alter erratically of itself, but only as the result of some variation in one or more of the energy-giving or absorbing forces, the error due to any variation of X in equation 51.2, when the other factors remain constant, is best regarded as an indirect error of the particular accidental change which caused the change of amplitude.

We shall therefore chassify the rate errors due to some accidental variation in connection with any particular force-application as follows :—*

(*a*) Primary Direct Error, because of the original change in the factor which has varied.

(*b*) Secondary Direct Errors, because that change modifies the amplitude and thereby affects the deviation due to this force in two ways :—

(*i*) Amplitude Error, because the amplitude is a factor of the deviation.

(*ii*) Timing Error, because the change of amplitude alters the phase corresponding to the point on the swing at which the impulse-timing signal is given.

(*c*) Indirect Errors due to similar effects of the change of amplitude on the deviation produced by every other force-application during the cycle.

As a rule, the cyclic changes in the driving force are much more complicated than a simple application and removal. Each such *change* of force is to be regarded as the ***application*** of an additional force, and each must be reckoned as a possible source of erratic variation.

It is only with certain simple types of escapement that it is possible to estimate the rate errors produced by any assumed erratic variation, and this we shall do in the remaining chapters.

(*To be continued.*)

* This classification is to be substituted for that previously given on page 161. Owing to an omission by the printer to make the changes marked on the proofs, the matter did not appear there as intended.

"Summer Time" Reminder. This year "Summer Time" begins at 2 a.m. on Sunday, April 13th, so that Easter comes after the change and not before, as was the case last year.

The Trade Charity. The National Benevolent Society of Watch and Clock Makers is badly in need of funds. An opportunity for special contributions is provided in connection with the Church Service which will be held at the City Temple, Holborn Viaduct, London, on Wednesday, June 18th, at 5.30, when the preacher will be the famous Rev. Dr. Horton. Please send contributions to the Secretary, Mr. James Savidge, F.C.I.S., 35, Northampton Square, London, E.C. 1.

(République Française) and le Directeur des Publications Officielles (Imprimerie Nationale), Paris, and the proprietors of the

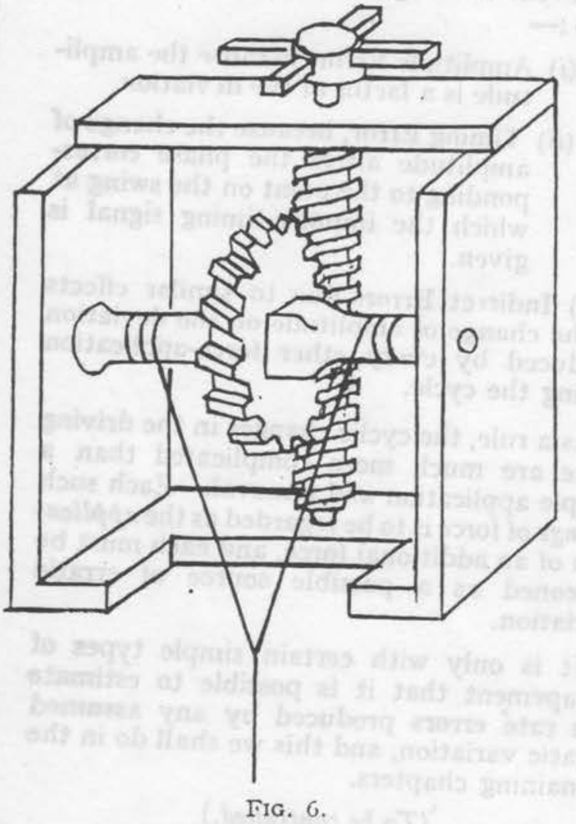

FIG. 6.

Revue Asiatique (Société Asiatique), I am now authorised to reproduce them. The drawings are extracts from an article entitled : "Les Mechaniques ou l'elevateur de Héron d'Alexandrie" (Mechanical Devices, or the Elevator of Hero of Alexandria), published for the first time in the Quostâ ibn Lugâ and translated (from Arabic) into French by Baron Cara de Vaux. I am very

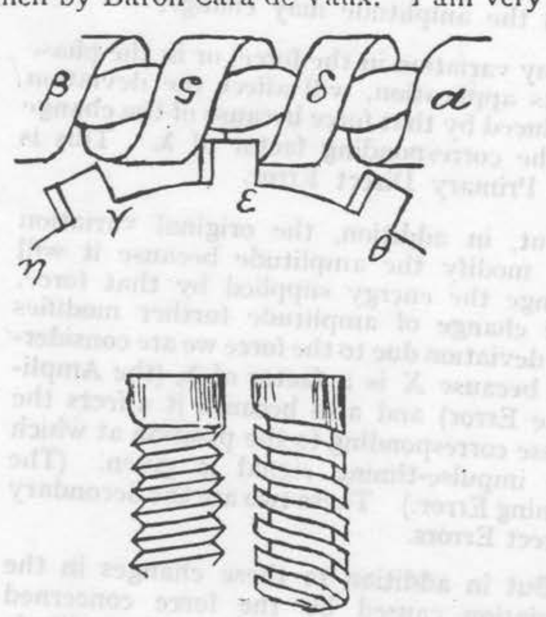

FIG. 7.

glad that this matter, after much research and correspondence, has been at last settled, and my best thanks have been forwarded to the parties concerned. This has appeared, as will be seen by lettering in Greek. My purpose, however, is only to illustrate Hero's Screws.—J. J. H.

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

(*Continued from page* 156.)

IX. Single-Phase Escapement Action.

52. *Single-Phase Escapement Action.*

A single-phase escapement is one in which a single constant force is applied at a fixed point on each leftward swing and removed at a fixed point on the return swing. If the application and removal take place at equal distances on opposite sides of the central position, that is at corresponding phases in the two swings, we have a symmetrical single-phase action. If the removal occur at some other point, the action is unsymmetrical.

If it is to supply energy to the vibration, the force must act towards the centre at the instant of its application (see section 34) ; in other words, it must be opposite to the displacement at that instant.

Another way of looking at the action is sometimes useful ; a single-phase escapement exerts a constant force on the pendulum, which force is reversed at a particular point in each swing. This idea is illustrated in the wave diagram of Fig. 45 and the vector diagram of Fig. 46.

Fig. 45 applies to an actual clock pendulum, with which the decay of the vibration during a single swing is too small to be visible, but in the vector diagram the force is very greatly exaggerated in order to make the displacement of the zero visible.

Moreover, damping has been left out of account in the latter diagram, with the result that the path of the vector-end is a series of semicircles, each larger than the previous ones, having their centres alternately at O′ and O″, at equal distances y on opposite sides of the true centre O.

In the actual case, the frictional resistance reduces the amplitude during the swing, and converts the semicircles into spirals; with steady running, the amplitude at each reversal is the same as at the previous one the same side.

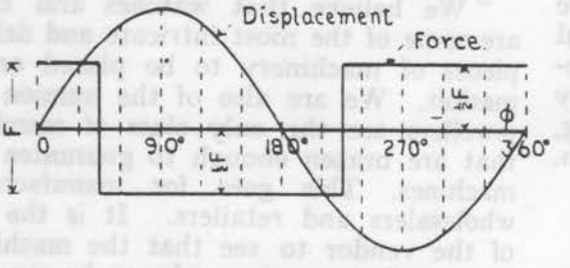

FIG. 45.—*Wave Diagram for a Symmetrical Single-Phase Escapement.*

The symmetrical single-phase action is the simplest of all possible ways of supplying the energy needed to keep the pendulum going against friction.

With such an escapement we are free to make our own selection of the following items:

(*a*) the shape of the pendulum, its size and material.

(*b*) the nature and density of the surrounding atmosphere.

(*c*) the running amplitude.

(*d*) the phase at which the force is reversed.

When these are fixed, the force to be employed is also fixed; for when reversed at the selected phase, it must supply energy equal to that absorbed by the resistance during one swing with the desired amplitude.

It is our task to so utilise our liberty of choice that the rate errors caused by unavoidable irregularities in the escapement action shall be a minimum.

There are three independent sources of such irregularities, namely:

(i) the driving force may vary in amount.

(ii) the phase of its application may vary.

(iii) the amplitude may change because of a variation in the frictional resistance to the motion of the pendulum.

With a given pendulum, our choice is restricted to the items (*b*), (*c*) and (*d*); it is not an unreasonably heavy task to make a full study of the effect of the selection made for each item on the probable rate errors due to each of the three prime causes.

With an unsymmetrical single-phase escapement, and also with a symmetrical

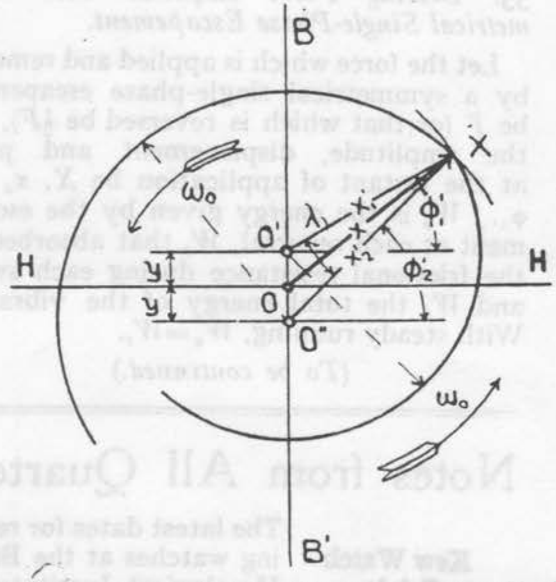

FIG. 46.—*Vector Diagram for a Symmetrical Single-Phase Escapement.*

two-phase one, there are two phases to be selected; the number of possible choices is thus doubled, and also the labour of investigating all possibilities.

With two-phase escapements having different actions on the two swings, the number is doubled again, and with polyphase escapements the freedom of choice may be so great that it is quite hopeless to attempt to analyse the results of all possible selections.

But any escapement action whatever may be duplicated by superimposing two or more single-phase escapements, each giving a definite fraction (some possibly negative) of the total energy required. A full analysis of the symmetrical single-phase escapement is consequently helpful in estimating the behaviour of any form of escapement.

But one point must not be overlooked. The secondary and indirect errors consequent upon some erratic change do not come into play at once, but only grow slowly as the amplitude builds up, or down, to its new value. This takes many cycles, and the change of amplitude during a single cycle is negligible. For these reasons, the resultant secondary and indirect errors are not always the same as the sum of those of the individual single-phase actions by which the given escapement action has been replaced.

The net change of amplitude must first be calculated, and then the various errors due to that change of amplitude may be added together.

53. *Driving Force Required with Symmetrical Single-Phase Escapement.*

Let the force which is applied and removed by a symmetrical single-phase escapement be F (or that which is reversed be $\frac{1}{2}F$), and the amplitude, displacement and phase at the instant of application be X, x_s and φ_s. W_e is the energy given by the escapement at each reversal, W_r that absorbed by the frictional resistance during each swing, and W_v the total energy of the vibration. With steady running, $W_e = W_r$.

(*To be continued.*)

Notes from All Quarters.

Kew Watch Trials. The latest dates for receiving watches at the British Horological Institute (35, Northampton Square, London, E.C. 1) for the next National Physical Laboratory Trials are Monday, 16th, and Monday, 30th, June, 1930.

◈

Guarantees. In Canada and the U.S.A. we notice that the question of guarantees for watches and their repair is exercising the minds of the jewellers, as apparently such guarantees are becoming more extended and extensive and so more difficult to stand by. We quote the following from a Canadian paper, *Trader* :—

"We want to say right here that with all his ingenuity, man has never yet been able to construct a machine that he can safely guarantee to run for any certain length of time. The thing is absurd. Machinery is, like those who made it, fallible, and at times the genius of man cannot find the defects in the machine he has created. This has happened with watches and clocks. So why guarantee them? Don't be foolish. True, there are times when the machines do run the guaranteed time. But it is those that do not fulfil the guarantee that cause the trouble. This goes for manufacturer, wholesaler and retailer.

"There is only one guarantee that any machine should carry. To protect the vendor and to be fair to the buyer, guarantee that your machine is as near perfect as possible when it is delivered. Then give service for a stated period of time."

It is urged that "service" has enabled the motor industry to develop its wonderful and effective selling system. Makers of cars and other kinds of machinery claim to turn out goods as nearly perfect as possible and back up their claims by giving a certain length of service. To quote again :—

"We believe that watches and clocks are some of the most intricate and delicate pieces of machinery to be placed on the market. We are also of the opinion that jewellers are the only class of merchants that are brazen enough to guarantee their machines. This goes for manufacturers, wholesalers and retailers. It is the duty of the vendor to see that the machine is in as perfect running order as he can make it at time of delivery."

The advice given by one retailer was :— "Do not guarantee your watches. Notify the purchaser to keep the watch regulated every few days." In other words give the purchaser service, say, for three months, and copy the motor car salesman. Of course absurd and impossible guarantees tend to commercial immorality, as some excuse has to be found for a charge after the watch has been in use some time. We should be glad to have the views of our readers on this important question.

◈

The British Watch and Clockmakers' Guild. We have received the 23rd Annual Report of this Society, the Annual Meeting of which is to be held on June 5th, at 268, St. John Street. The Guild has made a place for itself in the Horological world and is doing good work. Its finances are sound, as are its aims, and it has been able to assist the Trade Charity and also to offer prizes for practical work.

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By Professor DAVID ROBERTSON, D.Sc., M.I.E.E.

(*Continued from page* 193.)

From equation 34.09 we get :—

$F = -W_o/x_o = -W_o/X \sin \varphi_o$ . . (53.01)

With direct timing, the impulses occur at fixed values of x_o; the corresponding values of φ_o will depend upon the amplitude.

If, as in Fig. 46, the total displacement of the zero by the force F be $2y$, and we put this, instead of y, in equation 34.04, we get :—

$2y = F/k = -(W_r/kx_o)$ . . . (53.02)

From Fig. 15 we find that the pendulum G1, running in air at a pressure of 750 mm. with an amplitude of 100 arc-minutes, absorbs 79 ergs each swing. Table I gives the mass of this pendulum as 5,070 grams, and the distance down to its c.g. as 978 mm.

The linear amplitude with an angular amplitude of 100 arc-minutes is thus 28.4 mm., and the reversal should occur at a displacement of 70.7% of this, or 20.1 mm. if the phase of operation is to be 45°.

Hence the force to be applied at this phase is 79 ergs÷2.01 cm., or 39.4 dynes. This is the weight of 40 milligrams, or about 8 parts in a million of the weight of the pendulum.

Treating the vibration as a linear one, and omitting the suspension spring stiffness for the moment, the controlling constant is :—

$k = Mg/L = 5{,}070$ grams $\times 981$ (dynes/gram) $\div 978$ mm.

$= 5{,}085$ dynes per mm.

In Section 12 we estimated that the suspension spring would increase the rate by 600 s/d, or 0.7%. We may, therefore, suppose that it increases the controlling constant by twice this amount, raising k to 5,160 dynes per mm., and making the restoring force when the displacement is 20.1 mm. equal to 104 kilodynes.

The total displacement of zero required is thus 39.4 dynes÷5,160 (dynes per mm.), or 7.6 microns, which is 0.30 mils.

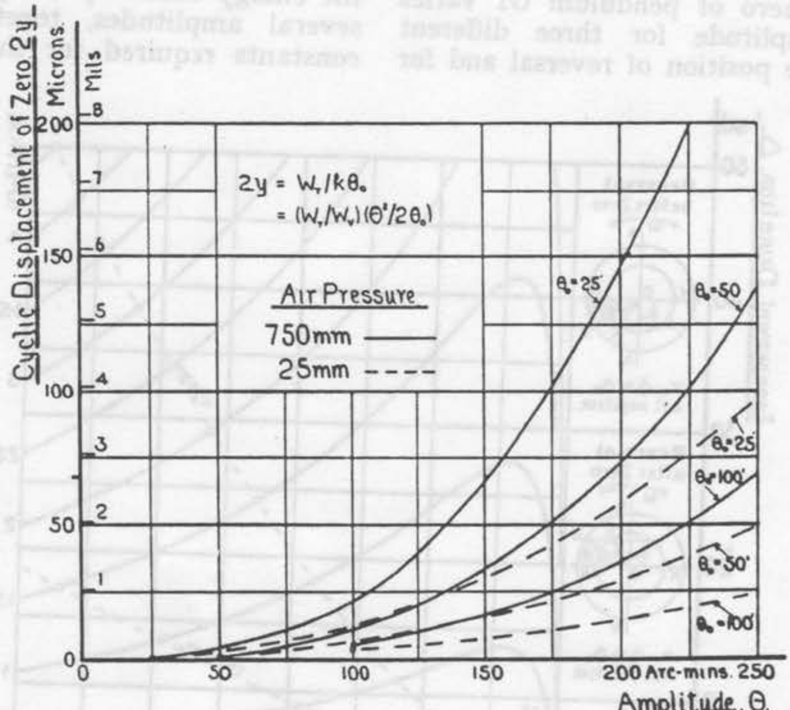

FIG. 47.—*Cyclic Displacement of Zero Required for Maintenance of Pendulum G1.*

The three curves of each set refer to reversals at displacements of 25, 50 and 100 arc-minutes.

It will be observed that the force to be applied is a very small one, and that the displacement of the zero is invisible.

Fig. 47 shows how the required displacement of the zero of pendulum G1 varies with the amplitude for three different settings of the position of reversal and for the two air pressures of 25 and 750 mm. These have been calculated in the manner just explained, from the data of Figs. 15 and 16, or of Table V, which gives the valves of the energy taken by the pendulum G1, at several amplitudes, together with other constants required for the calculation of

TABLE V.—ENERGY CONSTANTS OF PENDULUM G1.

Quantity	*Unit.*												
Air Press.	*mm.*	750						25					
Θ	Arc-mins.	50	100	150	200	250	300	50	100	150	200	250	300
W_r	Ergs.	13	79	241	540	1020	1720	11	46	112	214	358	550
W_v	10^3 ergs.	52	209	470	835	1300	1880	52	209	470	835	1300	1800
W_r/W_v	10^{-6}	245	378	514	648	782	916	201	219	238	256	275	293
q	10^{-3}	392	369	360	354	350	347	478	462	449	436	428	421
$q\ (W_r/W_v)$	10^{-6}	96	139	185	229	274	318	96	101	107	112	118	123

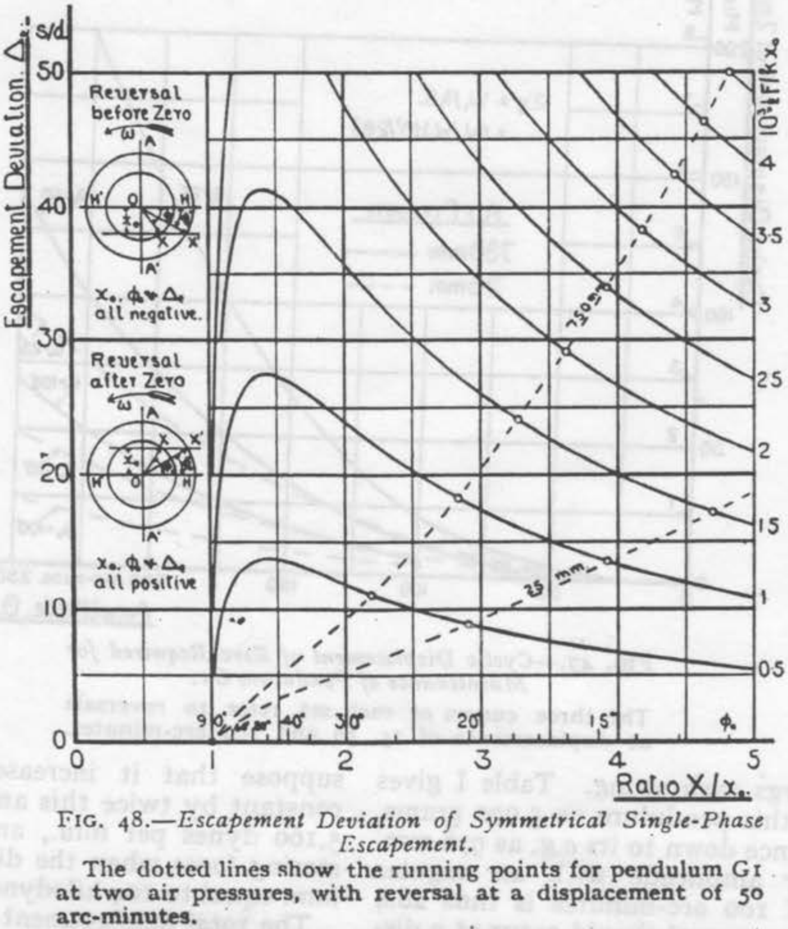

FIG. 48.—*Escapement Deviation of Symmetrical Single-Phase Escapement.*

The dotted lines show the running points for pendulum G1 at two air pressures, with reversal at a displacement of 50 arc-minutes.

The scale of degrees shows the phase at which reversal occurs.

the deviation and the errors due to assumed accidental variations.

54. *Deviation with Symmetrical Single-Phase Escapement.*

If the loss of phase at each reversal be λ, then since this loss takes place each *half*

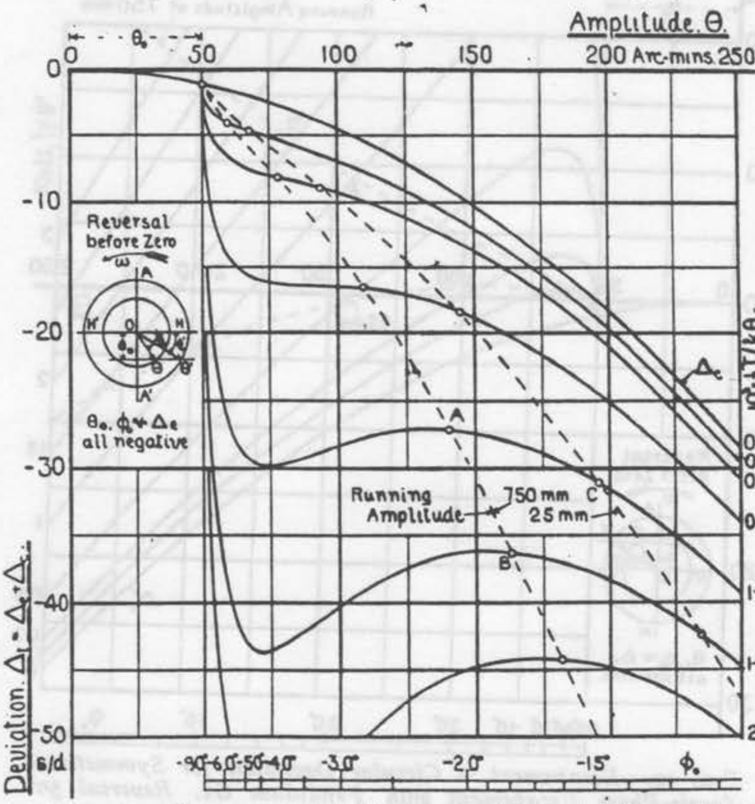

FIG. 49.—*Escapement + Circular Deviation for Symmetrical Single-Phase Escapement with Pendulum G1. Reversal 50′ before Zero.*

The dotted lines show the running points at two air pressures.

Δ_c is the circular deviation.

cycle, we get for the escapement deviation from equations 35.03—35.05 :—

$$\Delta_e = -\lambda/\pi = -(1/\pi)(F/kX)\cos\varphi_0 \quad (54.01)$$

$$= (1/\pi)(W_e/kX^2)\cot\varphi_0 \quad (54.02)$$

$$= (1/2\pi)(W_e/W_v)\cot\varphi_0 \quad (54.03)$$

Since $\sin\varphi_0 = x_0/X$, and $\cos\varphi_0 = (X^2 - x_0^2)^{\frac{1}{2}}/X$, equation 54.01 can be put into the form :—

$$\Delta_e = -(1/\pi)(F/kX)(X^2 - x_0^2)^{\frac{1}{2}}/X \quad (54.04)$$

$$= (1/\pi)(-F/kx_0)(x_0/X^2)\{(X/x_0)^2 - 1\}^{\frac{1}{2}} \quad (54.05)$$

$$= 27.5\times 10^8 s/d \times (-F/kx_0)(x_0/X)^2 \{(X/x_0)^2 - 1\}^{\frac{1}{2}} \quad (54.06)$$

and equation 54.03 can be written thus :—

$$\Delta_e = (1/2\pi)(W_e/W_v)(X^2 - x_0^2)/x_0 \quad (54.07)$$

It should be noted that $-kx_0$ is the controlling force acting at the point at which the escapement action takes place, and that $(-F/kx_0)$ is always positive. When the escapement forces are expressed as fractions of kx_0 and the amplitudes as multiples of x_0, the results will apply to any pendulum, but of course the actual running amplitude with a given force and a given value of x_0 will depend upon the resistance characteristic of the particular pendulum.

Fig. 48 shows how the escapement deviation varies with the amplitude for several values of $\frac{1}{2}F$ (the force which is reversed). All these curves are repetitions of the lowest one to different scales of ordinates.

The dotted lines show the running points with pendulum G1 at two air pressures if the escapement be set to operate at a displacement of 50 arc-minutes. These dotted lines would be higher if the escapement act further out, and lower if set to reverse nearer the centre.

The escapement deviation is negative when the reversal takes place before zero (i.e. during the inward movement) and positive when the action occurs after zero. The amount is the same in either case when the force and amplitude are the same. One set of graphs may, therefore, be employed for both.

With any other form of escapement, the escapement deviation graphs can be obtained as the sum of a number of curves such as those of Fig. 48. But as the component curves will have their peaks at different amplitudes, have probably different values for these peaks, and have possibly positive deviations in some cases and negative ones in others, the resultant graph may have no resemblance whatever to that of the single-phase escapement.

The circular deviation is always negative; the sum of the escapement and circular deviations is consequently different with action before and after zero. Figs. 49 and 50 give this sum for these two cases for pendulum G1 operated by a symmetrical single-phase escapement reversed at a displacement of 50 arc-minutes. As these graphs definitely refer to a vibration about an axis, 0, Θ and τ have been used instead of x, X and F.

Two points of considerable importance are illustrated by these figures. The first is that the error due to a variation of amplitude depends upon the cause of that variation. The running point moves along one of the dotted lines when the driving force is varied without change of the pendulum resistance characteristic, but along one of the full lines when the resistance alters whilst the driving force remains constant.

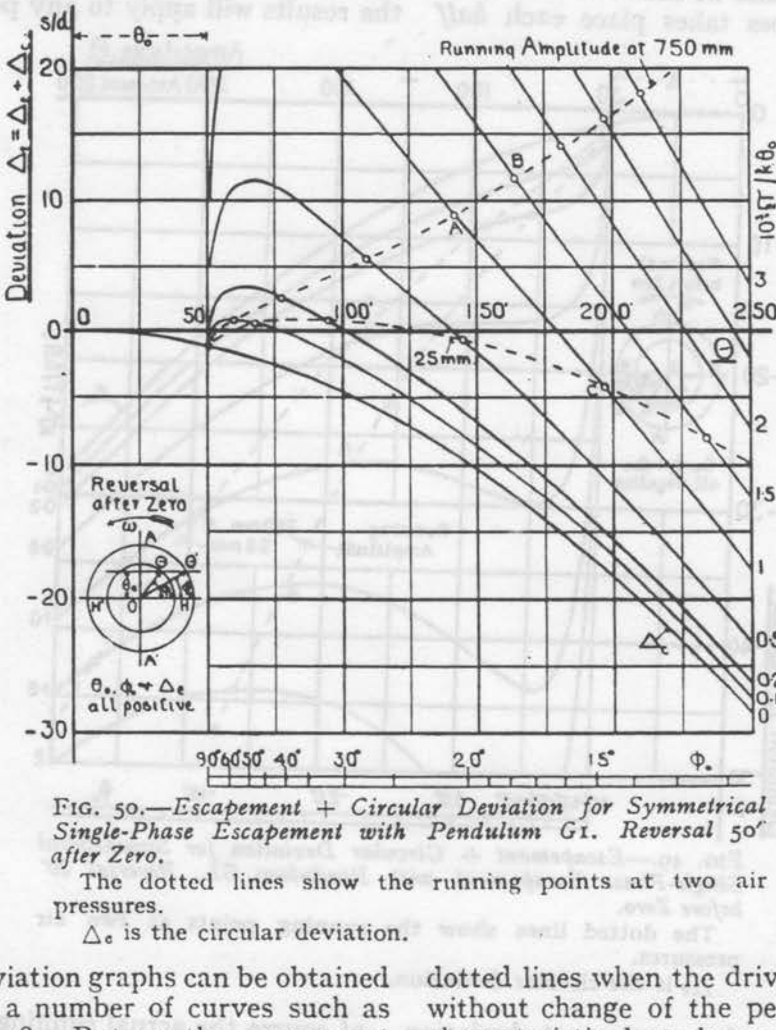

FIG. 50.—*Escapement + Circular Deviation for Symmetrical Single-Phase Escapement with Pendulum G1. Reversal 50′ after Zero.*

The dotted lines show the running points at two air pressures.

Δ_c is the circular deviation.

Thus, starting with the point *A* in either figure, a 50% increase of driving force alone will send the running point to *B*, but a reduction of air pressure from 750 to 25 mm. with the driving force constant would transfer the running point from *A* to *C*.

In either case the difference in level between the final and initial points gives the escapement plus circular error due to the change; with change of air pressure, there is also the air buoyancy and inertia error to be allowed for.

(*To be continued.*)

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

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(*Continued from page 222.*)

The other point is that a condition which gives zero deviation is not necessarily a desirable one to select for running. With the reversals occurring at the end of the swing (when $\varphi_0=90°$ and $X=x_0$) there is no escapement deviation. But a small drop of amplitude would put the escapement out of action altogether, and a small increase would give a very large escapement error because of the great steepness of the deviation curve at the place selected.

What is required is to select a running point at a place where the curve is as nearly horizontal as possible. Thus, points *A* and *B* on Fig. 49 are both good running points so far as variation of resistance is concerned; in fact, the 750 mm. dotted line cuts the full beginning of the curve where it is a little, but not much, less steep. But with reversal after zero (Fig. 50) the low-pressure curve has a flat part and a place can be found where neither the full nor the dotted curve is a very steep.

A similar result holds at atmospheric pressure provided that θ_0 exceeds a critical value of about 90′. But the horizontal parts of the full and dotted lines occur at different amplitudes, and so we cannot simultaneously have the best conditions for both kinds of variation.

This difficulty, that the best conditions for one sort of variation are not also the best for another sort, always crops up, and it is necessary to make some sort of compromise.

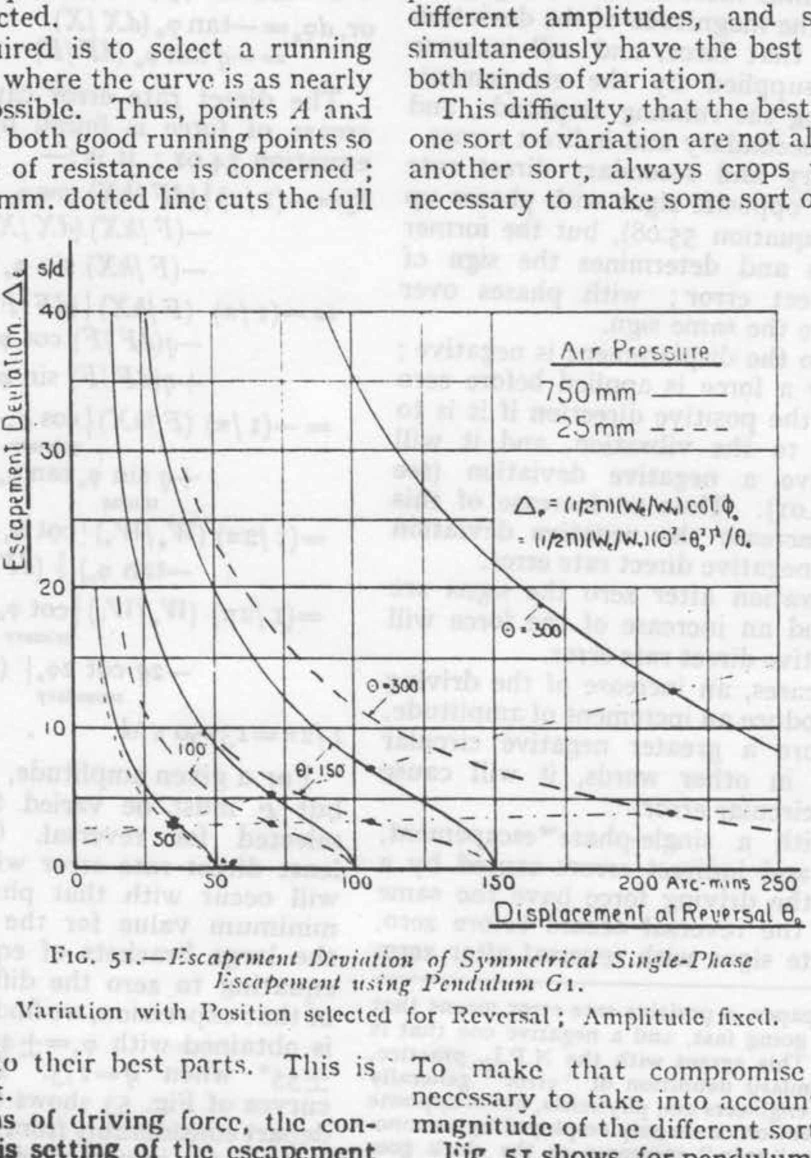

FIG. 51.—*Escapement Deviation of Symmetrical Single-Phase Escapement using Pendulum G1.*

Variation with Position selected for Reversal; Amplitude fixed.

lines all close to their best parts. This is a mere accident.

For variations of driving force, the conditions with this setting of the escapement are not so good; the best point is near the

To make that compromise wisely it is necessary to take into account the probable magnitude of the different sorts of errors.

Fig. 51 shows, for pendulum G1, the variation of the escapement deviation with the position selected for the reversal of the force. Here again all the curves are similar, the abscissæ being proportional to the amplitude to which each refers, and the ordinates to the value of (W_r/W_v) for that amplitude. Chain dotted lines have been added through the points corresponding to phase 45°.

The curves apply to reversal after zero; with the sign reversed, they would apply to operation before zero. They do not include the circular deviation, which is negative in both cases, and shifts the curves downwards by a definite amount for each amplitude, namely 1.14, 10.28 and 41.4 s/d for amplitudes of 50, 150 and 300 arc-minutes.

55. *Rate Errors due to Variation of Driving Force.*

Any accidental increase of driving force will increase the magnitude of the deviation produced by that force, and will increase the energy supplied by the escapement, thereby raising the running amplitude, and thus causing secondary and indirect errors.

The primary and secondary direct rate errors * have opposite signs with phases up to 45° (see equation 55.08), but the former predominates and determines the sign of the total-direct error; with phases over 45° they have the same sign.

Before zero the displacement is negative; consequently a force is applied before zero must act in the positive direction if it is to give energy to the vibration, and it will therefore give a negative deviation (see equation 54.01). Thus an increase of this force will increase the negative deviation and cause a negative direct rate error.

With operation after zero the signs are reversed, and an increase of the force will cause a positive direct rate error.

In both cases, an increase of the driving force will produce an increment of amplitude, and therefore a greater negative circular deviation; in other words, it will cause a negative circular error.

Thus, with a single-phase escapement, the direct and indirect errors caused by a change in the driving force have the same sign when the reversal occurs before zero, but opposite signs with reversal after zero.

* In this paper, a positive rate error means that the clock is going fast, and a negative one that it goes slow. This agrees with the N.P.L. practice, and the standard definition of "error" generally accepted by engineers and physicists, but is opposite to the convention commonly employed by astronomers, whose "rate" increases as the clock goes slower.

A reference to Figs. 53-55 will show that in the latter case it is possible to so choose the conditions that the direct and indirect errors balance one another, with the excellent result that a small change of driving force does not alter the rate at all.

Let us suppose that the driving force F is increased by the small amount dF, increasing W_e, the energy supplied by the escapement, by the amount dW_e, and the amplitude by the amount dX. Since x_0 is not affected by a change in the force, it follows from equation 34.09 that the energy and force are proportional to one another. Hence we may write equation 16.02 in the form:—

$$dX/X = q\,(dW_e/W_e) = q\,(dF/F) \quad . \quad (55.01)$$

Also, since $x_0 = X \sin\varphi_0$, we have:—

$$O = dX \sin\varphi_0 + X\cos\varphi_0\, d\varphi_0 \quad . \quad (55.02)$$

$$\text{or, } d\varphi_0 = -\tan\varphi_0\,(dX/X)$$
$$= -q\tan\varphi_0\,(dF/F) \quad . \quad (55.03)$$

The direct rate error caused by this increase of force is found by differentiating equation 54.01; it is:—

$$\delta_e = -(1/\pi)\{(dF/kX)\cos\varphi_0 - (F/kX)(dX/X)\cos\varphi_0 - (F/kX)\sin\varphi_0\, d\varphi_0\} \quad . \quad (55.04)$$

$$= -(1/\pi)\,(F/kX)\{(dF/F)\cos\varphi_0 - q(dF/F)\cos\varphi_0 + q(dF/F)\sin\varphi_0\tan\varphi_0\} \quad (55.05)$$

$$= -(1/\pi)\,(F/kX)\{\underset{\text{primary}}{\cos\varphi_0} - \underset{\text{amplitude}}{q\cos\varphi_0} + \underset{\text{timing}}{q\sin\varphi_0\tan\varphi_0}\}\,(dF/F) \quad (55.06)$$

$$= (1/2\pi)\,(W_e/W_v)\{\cot\varphi_0 - q(\cot\varphi_0 - \tan\varphi_0)\}\,(dF/F) \quad . \quad (55.07)$$

$$= (1/2\pi)\,(W_e/W_v)\{\underset{\text{primary}}{\cot\varphi_0} - \underset{\text{secondary}}{2q\cot 2\varphi_0}\}\,(dF/F) \quad . \quad (55.08)$$

$$1/2\pi = 13750\ s/d \quad . \quad . \quad . \quad (55.09)$$

For a given amplitude, W_e and q are fixed, but F must be varied to suit the phase selected for reversal. Consequently, the least direct rate error with that amplitude will occur with that phase which gives a minimum value for the expression within the large brackets of equation 55.08. By equating to zero the differential coefficient of that expression, we find that the minimum is obtained with $\varphi_0 = \pm 45°$ when $q = \frac{1}{2}$, and $\pm 55°$ when $q = 1/3$. The shape of the curves of Fig. 53 shows that the phase may depart considerably from these values without making any important difference to the error.

When φ_0 lies between the limits $\pm 45°$, cot 2 φ_0 is smaller than cot φ_0; since $2q$ never exceeds unity (see section 16), the secondary error is always smaller than the primary within these limits of φ_0. Outside these limits, up to 90°, which is the greatest value φ_0 can have, the two cotangents have opposite signs and the primary and secondary errors have the same sign. Thus, in either case, the total direct error due to variation of the driving force has the same sign as its primary component.

We obtain the circular error from equation 37.15 (where Θ is used instead of X for the amplitude because circular deviation does not occur with linear vibrations), thus :—

$$\delta_c = -(1/8)\ \Theta^2\ (d\ \Theta/\Theta)$$

$$= -(q/8)\ \Theta^2\ (dF/F) \qquad (55.10)$$

$$= -9.14\ s/d\ q(\Theta/100 \text{ arc-minutes})^2\ (dF/F) \qquad (55.11)$$

Observe that each component of the direct error depends upon the phases selected for the reversal of the force, and is proportional to the ratio (W_e/W_v) of the energy supplied each swing to the total energy of the vibration. The secondary component is also proportional to the ratio q = proportional increase of the amplitude ÷ proportional increase of the energy required. The direct error depends upon the amplitude only in so far as that affects these two ratios, which may be obtained from Table V.

On the other hand, the circular error does not depend upon the selected phase in this case, nor upon the ratio W_e/W_v, but it is proportional to the ratio q and to the square of the amplitude.

Fig. 52 shows the values of these three ratios for pendulum G1 at two air pressures.

(*To be continued.*)

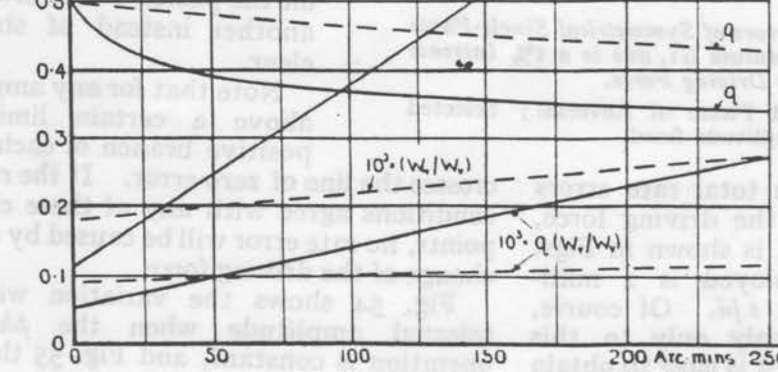

FIG. 52.—*Energy Ratios for Pendulum G1.*

Department of Overseas Trade. The Comptroller-General of the Department of Overseas Trade sends the following information :

A memorandum on the Department Stores Trade in the Pacific North-West of the United States of America has been received from the Acting British Consul-General at San Francisco.

United Kingdom firms desirous of receiving a copy of the memorandum should apply to the Department of Overseas Trade, 35, Old Queen Street, London, S.W. 1, quoting reference No. C.X. 3300.

CANADA—OVEN THERMOMETERS.

His Majesty's Trade Commissioner at Toronto reports that an Ontario firm manufacturing electric stoves is desirous of importing oven thermometers.

United Kingdom manufacturers of these instruments can obtain the names of the enquirers on application to the Department of Overseas Trade, 35, Old Queen Street, London, S.W. 1. Reference number B.X. 6700 should be quoted.

MARKET FOR FINE AND IMITATION JEWELLERY IN ARGENTINA.

A confidential report on the market for fine and imitation jewellery in Argentina has been prepared by the Department of Overseas Trade from information furnished by the Commercial Counsellor, His Majesty's Embassy, Buenos Aires, and issued to firms whose names are entered on its Special Register.

United Kingdom manufacturers of jewellery desirous of receiving a copy of this report together with particulars of the Special Register service of information and form of application for registration should communicate with the Department of Overseas Trade, 35, Old Queen Street, London, S.W. 1. Reference number B.X. 6687 should be quoted.

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

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(*Continued from page 18.*)

It will be observed that above a certain amplitude the two ratios which affect the direct errors are reduced by adopting the lower air pressure, but that the third ratio q, to which the circular errors are proportional, is increased. Whether the total error is reduced or increased will thus depend upon the relative values of the direct and indirect errors.

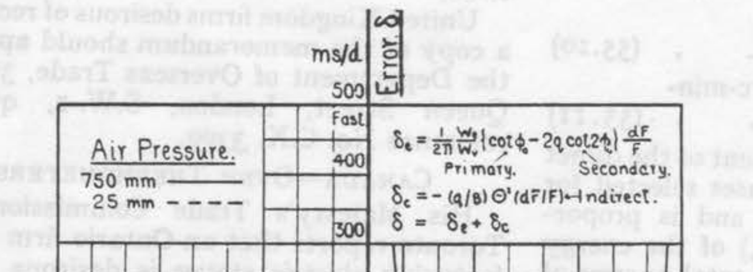

FIG. 53.—*Total Rate Errors of Symmetrical Single-Phase Escapement, using Pendulum G1, due to a 1% Increase of the Driving Force.*

Variation with Selected Phase of Reversal; Selected Amplitude fixed.

The magnitude of the total rate errors due to a 1% increase of the driving force, when using pendulum G1, is shown in Figs. 53–55. The unit employed is 1 millisecond per day = 0.001 s/d. Of course, the numerical values apply only to this particular pendulum, but it is easy to obtain similar figures for any other pendulum run under stated conditions. The difference between one pendulum and another of about the same weight and run under the same conditions would probably be less than that between the two conditions given for this pendulum. Lighter pendulums would have a greater (W_s/W_v), which would magnify the direct errors, leaving the circular error much about the same. This would raise the curves for positive phases and lower those for negative phases.

Fig. 53 shows the variation with the selected phase when the amplitude is fixed, values for two air pressures being given. Since the circular error is constant for a given amplitude, the shape of the curves is determined by the direct errors only. With operation at equal distances on opposite sides of zero, the direct errors are equal and opposite; consequently, they are given by the ordinate measured from the horizontal line drawn midway between the two branches of the curve considered.

But the circular error destroys the symmetry by shifting all the curves downward to an extent which increases with the amplitude, and with diminished air pressure. The shift is sufficient to make the curves on the positive side cross one another instead of standing clear.

Note that for any amplitude above a certain limit, the positive branch of each twice crosses the line of zero error. If the running conditions agree with any of these crossing points, no rate error will be caused by a small change of the driving force.

Fig. 54 shows the variation with the selected amplitude when the *phase* of operation is constant, and Fig. 55 the same variation (at 750 mm.. only) with a fixed *position* of reversal. The latter is the more useful, for different points on one graph correspond to different driving forces with the same setting of the escapement.

In Fig. 55 the graphs for all positive settings beyond a critical one cut the axis of zero error; also, the nearer the setting is to the critical point, the more horizontal the line at the crossing point. This latter is an advantage, for it means that the effect of inaccurate adjustment to the crossing point is small.

The crossing point for any setting can be found by direct experiment; it is only necessary to observe the rate with different driving forces. The

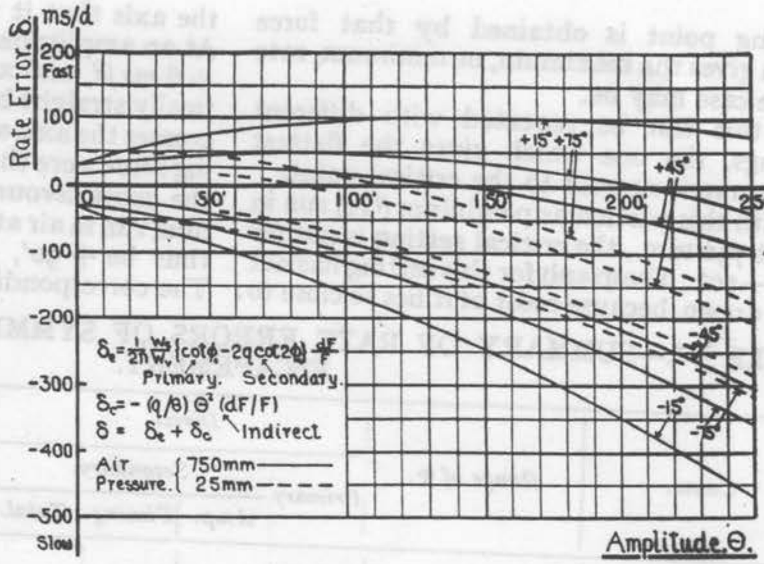

FIG. 54—*Total Rate Errors of Symmetrical Single-Phase Escapement, using Pendulum G1, due to a 1% Increase of the Driving Force.*

Variation with Selected Amplitude; Selected Phase of Reversal fixed.

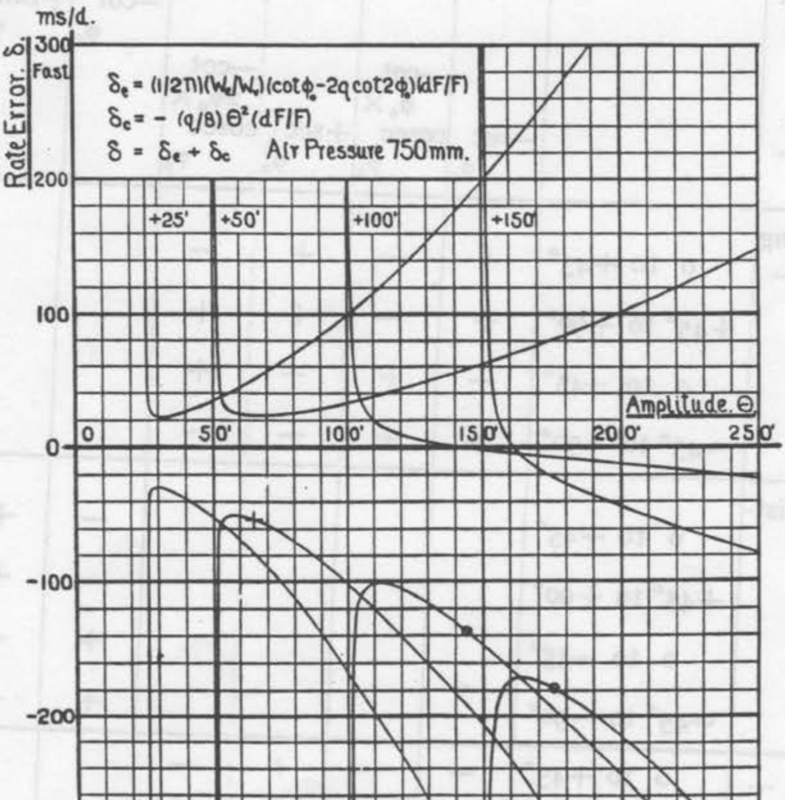

FIG. 55.—*Total Rate Errors of Symmetrical Single-Phase Escapement, using Pendulum G1, due to a 1% Increase of the Driving Force.*

Variation with Selected Amplitude; Selected Position of Reversal fixed.

crossing point is obtained by that force which gives the maximum, or minimum, rate as the case may be.

If this test be repeated with different settings, the one which gives the flattest rate curve is nearest to the critical point.

With this particular pendulum (G1) run in air at 750 mm., the critical setting is not far from $+90'$. The graph for this setting has not been drawn, because most of it lies so close to the axis that it would not show distinctly. At an amplitude of 150′ it gives an error of $+6$ *ms/d*, and at 250′ -4 *ms/d*. It is practically straight between these two limits and crosses the axis at 210′. If variations of driving force were the only thing to be considered, the most favourable setting for this pendulum, run in air at atmospheric pressure, would thus be $+90'$, with an amplitude of 210′. The corresponding phase is $+25°$.

TABLE VI.—SUMMARY OF RATE ERRORS OF SYMMETRICAL SINGLE-PHASE ESCAPEMENT.

Cause.	*Range of* Φ.	*Direct.*				*Indirect.*			
		Primary	*Secondary.*			*Ampl.*	*Timing.*	*Amp. + Timing.*	*Circular*
			Amp.	*Timing.*	*Total.*				
Increase of driving force		$+\cot\varphi_0$	$-\cot\varphi_0$	$+\tan\varphi_0$	$-\cot 2\varphi_0$				
Decrease of resistance						$-\cot\varphi_0$	$+\tan\varphi_0$	$-\cot 2\varphi_0$	Does not vary with φ_0
Reversal late ...		$-\sec\varphi_0$	$-\cot\varphi_0 \times \operatorname{cosec}\varphi_0$	$+\sec\varphi_0$	$-\cot 2\varphi_0 \times \operatorname{cosec}\varphi_0$				$-\operatorname{cosec}\varphi_0$
Increase of driving force	0 to $+45°$	+	−	+	−				−
	$+45°$ to $+90°$	+	−	+	+				−
	0 to $-45°$	−	+	−	+				−
	$-45°$ to $-90°$	−	+	−	−				−
Decrease of resistance ...	0 to $+45°$					−	+	−	−
	$+45°$ to $+90°$					−	+	+	−
	0 to $-45°$					+	−	+	−
	$-45°$ to $-90°$					+	−	−	−
Reversal late ...	0 to $+45°$	−	−	+	−				−
	$+45°$ to $+90°$	−	−	+	+				−
	0 to $-45°$	−	−	+	−				+
	$-45°$ to $-90°$	−	−	+	+				+

(*To be continued.*)

the members of the trade to support their action in announcing five vacancies for out pensions (three male and two female).

The Society finds it a constant struggle to make both ends meet. New subscribers are urgently required. The depressed state of trade should spur the charitably disposed in the trade, and there must be many, to make a point of supporting their own trade charity. There are many reasons for this.

It is one of the oldest craft charities in existence.

Its works and benefits are national and comprehensive.

The Society is managed by horologists for horologists.

Since the amalgamation it is the only charity for watch and clockmakers.

The Society needs new subscribers badly.

THE ELECTION FOR FIVE OUT PENSIONS.

The election for five out pensioners will take place in May.

Intending candidates can obtain form of application from the Secretary at the Office, 35, Northampton Square, London, E.C. 1.

Candidates are asked to attend before the Committee on Tuesday, March 31st, at 5.30 p.m.

For further information apply to the Secretary.

THANKSGIVING SERVICE AT ST. PAUL'S CATHEDRAL.

The 1931 Thanksgiving Service will take place at St. Paul's Cathedral on Tuesday, June 23rd, at 5.30 p.m.

The Committee have pleasure in announcing that the Right Rev. the Lord Bishop of London will preach the sermon and make an appeal on behalf of the funds.

Members of the watch and clock trade should feel proud to think that the authorities of the great Metropolitan Cathedral have honoured them by granting the use of the building for this service.

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By PROFESSOR DAVID ROBERTSON, D.Sc., M.I.E.E.

(*Continued from page* 54.)

Fig. 56 shows, to a larger scale, the various components of the errors when the selected phase is $+ 45°$. At this phase, $\cot \varphi_0 = \tan \varphi_0$, and the two components of the secondary error balance one another. With operation nearer to the centre (φ_0 smaller), the negative amplitude-component of the secondary error will exceed the positive timing-component, but with reversal further out the reverse will be true.

The crossing point with phase $+ 45°$ occurs at an amplitude of 148 arc-minutes at the higher air pressure, and at 82 arc-minutes with the lower pressure.

The figures of Table V refer to the pendulum only. The addition of a crutch, or other cause of friction, will increase W_r by the amount of the energy absorbed by that friction, and make q very slightly greater. The primary errors will be increased in the same ratio as W_r, the secondary errors in a somewhat greater ratio, and the circular errors by a small amount. The graphs would thus be altered in shape, and the crossing points come considerably different.

56. *Rate Errors due to Variation of Position of Reversal.*

The phase of the escapement action is determined by the pendulum reaching a particular point on its swing at which the escape wheel is unlocked, the electric contact made, or in some other way the release of the energy is controlled.

Variations of the phase of the action may be brought about by wear of the pallets and teeth, or of the electric contacts, by end-play combined with obliquity of the faces, by slackness and wear of the bearings, and by differential expansion of the parts which locate the two elements whose contact controls the action. With electric control, a further cause of variation of the phase of action arises from the variable time taken by the armature to make its stroke.

The wear of sliding pallets causes the action to take place sooner, but that of electric contacts makes it later.

Let the displacement at which the action should occur be x_0, and suppose that one of the causes mentioned delays it until $(x_0 + dx_0)$.

The change of x_o varies φ_o and therefore causes a primary direct error in the rate. But it also alters the energy supplied to the pendulum and thereby affects the running amplitude, which in its turn causes a secondary direct error due to a further change of phase, and indirect errors by varying the deviations due to any other forces, such as those causing circular deviation.

$$\delta_e = (1/\pi)(F/kX)\{\sin\varphi_o\, d\varphi_o + \cos\varphi_o\,(dX/X)\} \quad (56.07)$$

$$= (1/\pi)(F/kX)\{\underset{\text{primary timing}}{(1} - \underset{\text{secondary timing}}{q)\tan\varphi_o} + \underset{\text{amplitude}}{q\cot\varphi_o}\}(dx_o/X) \quad (56.08)$$

$$= (1/\pi)(F/kX)\{\tan\varphi_o - q(\tan\varphi_o - \cot\varphi_o)\}(dx_o/X) \quad (56.09)$$

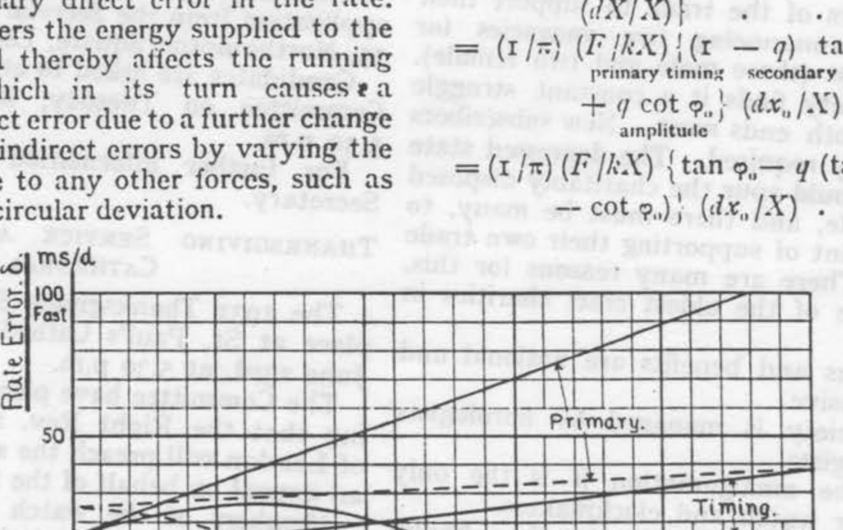

FIG. 56.—*Rate Errors of Symmetrical Single-Phase Escapement, using Pendulum G1, due to a 1% Increase of the Driving Force reversed at Phase +45°.*

Equation 34.09 gives us :—

$$W_e = -FX\sin\varphi_o = -Fx_o \quad (56.01)$$

$$\therefore dW_e = -F dx_o \quad (56.02)$$

Also, from equation 16.02 we have :—

$$(dX/X) = q(dW_e/W_e) = q(dx_o/x_o) \quad (56.03)$$

Again, since $\sin\varphi_o = x_o/X$, we have :—

$$\cos\varphi_o\, d\varphi_o = dx_o/X - (x_o/X)(dX/X) \quad (56.04)$$

$$= (1 - q)(dx_o/X) \quad (56.05)$$

$$\therefore d\varphi_o = (1-q)\sec\varphi_o\,(dx_o/X) \quad (56.06)$$

The direct escapement error due to a variation of φ_o is obtained by differentiating equation 54.01, treating F as constant, but X variable as well as φ_o, thus :—

$$= (1/\pi)(F/kX)\{\tan\varphi_o + 2q\cot 2\varphi_o\}(dx_o/X) \quad (56.10)$$

$$= -(1/2\pi)(W_e/W_v)\{\tan\varphi_o + 2q\cot 2\varphi_o\}(dx_o/x_o) \quad (56.11)$$

$$= -(1/2\pi)(W_e/W_v)\{\underset{\text{primary}}{\sec\varphi_o} + \underset{\text{secondary}}{2q\,\mathrm{cosec}\,\varphi_o\cot 2\varphi_o}\}(dx_o/X) \quad (56.12)$$

As before, we can find the phase of operation which will give the least direct error by equating to zero the differential of the expression inside the large brackets. Doing so, we obtain $54\frac{3}{4}°$ when $q = \frac{1}{2}$ and $48\frac{1}{2}°$ when $q = 1/3$.

(*To be continued.*)

THE THEORY OF PENDULUMS AND ESCAPEMENTS.

By PROFESSOR DAVID ROBERTSON, D.Sc., M.I.E.E.

(*Continued from page* 130)

The circular error is :—

$$\delta_c = -(1/8)\ \Theta^2\ (d\ \Theta/\Theta) \quad . \quad . \quad (56.13)$$

$$= -(q/8)\quad \Theta^2 \quad (d\theta_o/\theta_o) \quad . \quad . \quad (56.14)$$

$$= -(q/8 \operatorname{cosec} \varphi_o\ \Theta^2\ (d\theta_o/\Theta) \quad . \quad (56.15)$$

With a given amplitude, the circular error due to a given change in the position of the pendulum at the instant of reversal of the force does vary with the phase selected for that reversal, because the energy given is altered in the same ratio as θ_o. But if the error in the position be a fixed fraction of θ_o, instead of a fixed amount, it would not do so. The former is the more probable sort of variation and has been chosen for the graphs.

The magnitude of the total errors caused by the pendulum being 1 arc-minute further on than the correct position is shown in Figs. 57–60 for pendulum G1 at two air pressures. With a pallet or contact leverage of 1 inch, 1 arc-minute corresponds to 0.291 mils.

For this variation, the direct rate errors at equal phases on opposite sides of zero are again equal in magnitude, but now they are both negative. On the other hand, the circular error is positive with operation before zero, and negative after zero, because $d\theta_o$ reduces the distance from the centre in the former case and increases it in the latter. Thus the two errors have the same sign

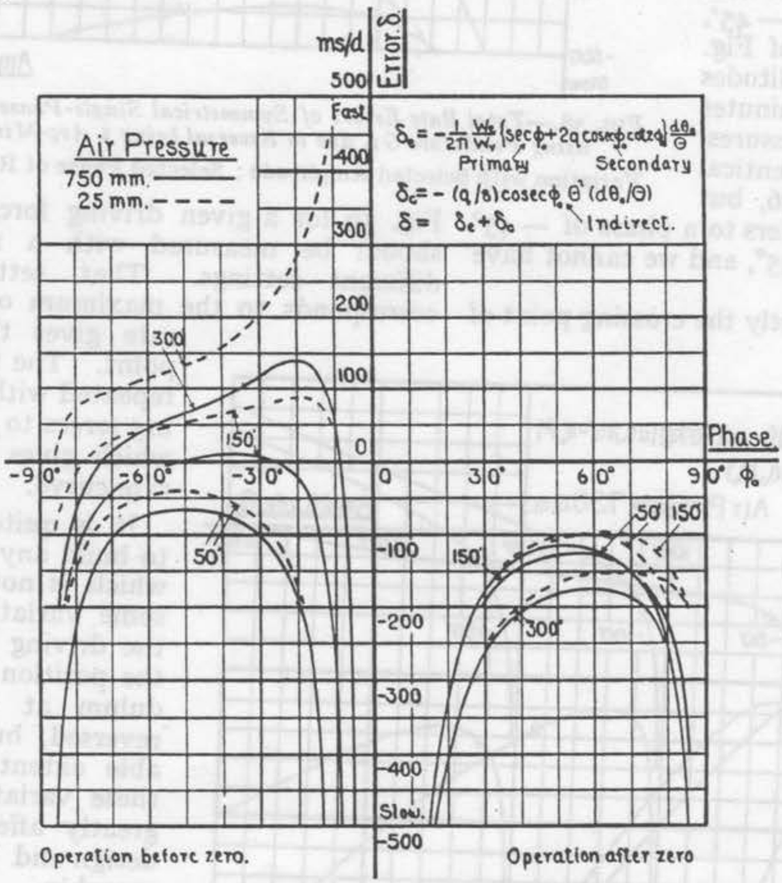

FIG. 57.—*Total Rate Errors of Symmetrical Single-Phase Escapement, using Pendulum G1, due to Reversal being 1 Arc-Minute late.*

Variation with Selected Phase of Reversal; Selected Amplitude fixed.

with operation after zero and opposite signs before zero, which is just the reverse of what applies to variations of the driving force.

A full discussion of these four sets of graphs is hardly necessary, for, after allowing for the fact that it is now the negative branches of the curves which cut the line of zero error, it would be a mere repetition of that of the previous set of four.

But the following points may be noted. The critical setting is now about $-45'$. The crossing points of Fig. 60 occur with amplitudes of 144 and 85 arc-minutes for the two air pressures. These are almost identical with those of Fig. 56, but unfortunately one refers to a phase of $-45°$ and the other to $+45°$, and we cannot have them both at once.

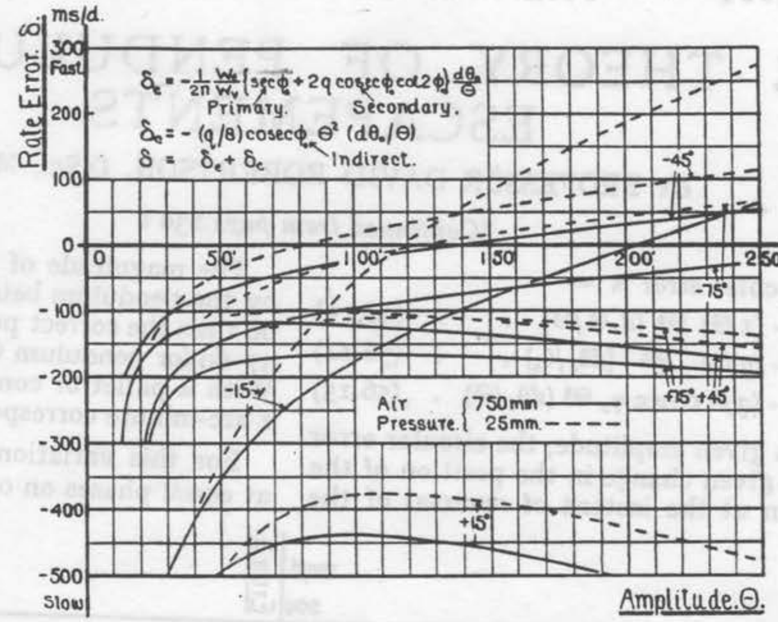

FIG. 58.—*Total Rate Errors of Symmetrical Single-Phase Escapement, using Pendulum G1, due to Reversal being 1 Arc-Minute late.*

Variation with Selected Amplitude ; Selected Phase of Reversal fixed.

To determine directly the crossing point of Fig. 59 for a given driving force, the rate should be measured with a number of different settings. That setting which corresponds to the maximum or minimum rate gives the crossing point. The test may be repeated with other driving forces to find the one which gives the flattest rate curve.

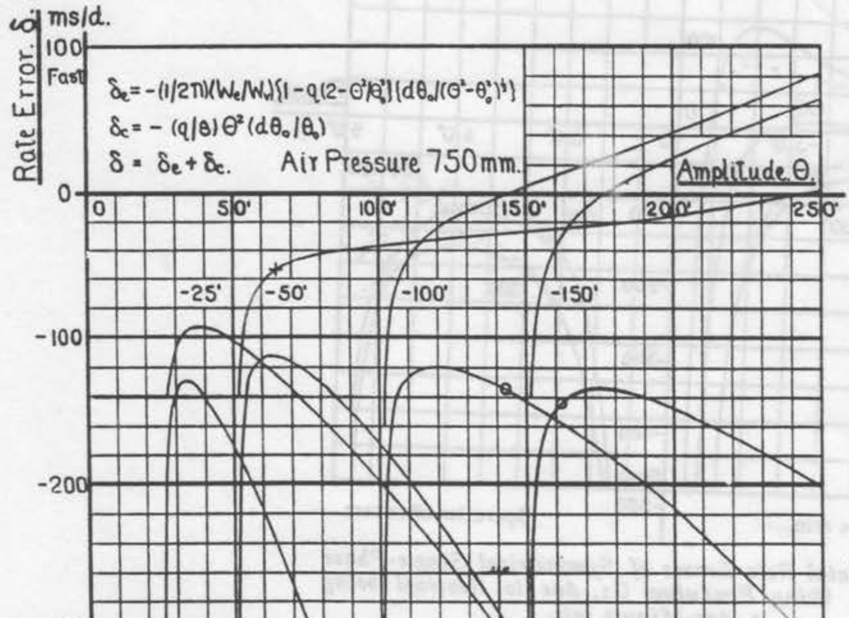

FIG. 59.—*Total Rate Errors of Symmetrical Single-Phase Escapement, using Pendulum G1, due to Reversal being 1 Arc-Minute late.*

Variation with Selected Amplitude ; Selected Position of Reversal fixed.

It is quite impossible to build any escapement which is not subject to some variation both of the driving force and of the position of the pendulum at which it is reversed, but the probable extent of each of these variations will be greatly affected by the design and by the workmanship.

If, with a given mechanism, the probable erratic variations of driving force are much more serious than those of the position at reversal, we should employ a positive setting and select conditions agreeing with one of the crossing points of

Fig. 55. But if the probable importance of the two variations be reversed, we should prefer a negative setting and a crossing point of Fig. 59. A negative setting is not possible with all types of escapement.

(To be continued.)

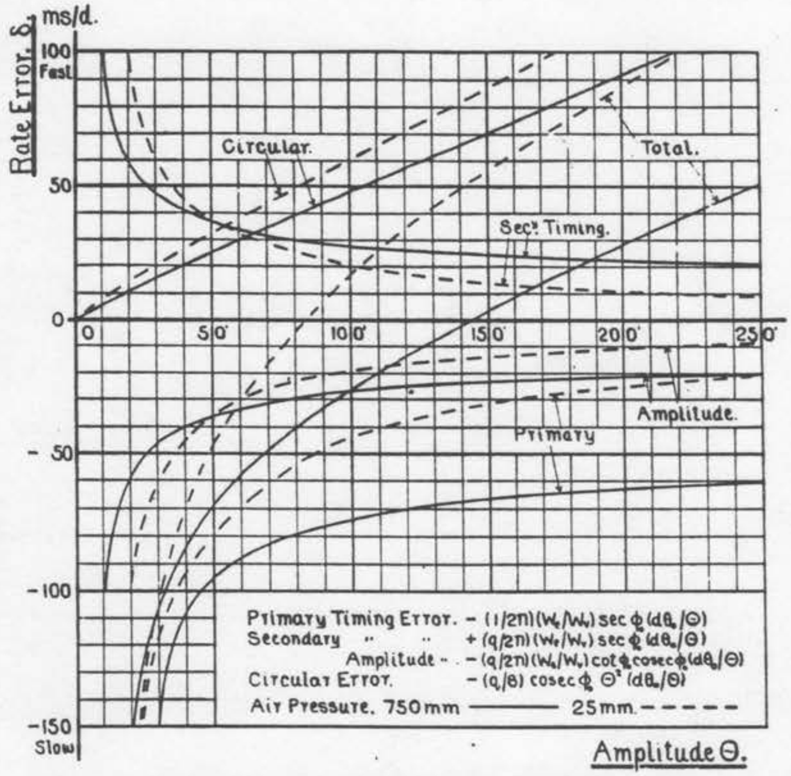

FIG. 60.—*Rate Errors of Symmetrical Single-Phase Escapement, using Pendulum G1, due to Reversal being 1 Arc-Minute late, when the Sected Phase is −45°.*

PENDULUM CLOCKS AND GRAVITY VARIATION

BY W. BOWYER.

At the last meeting of the Royal Astronomical Society two papers were communicated, giving the results of an investigation on the above subject. The experiments were conducted at the Laboratory of Mr. A. L. Loomis, at Tuxedo Park, New York, U.S.A., the object being to test whether observatory precision clocks were of sufficient accuracy in their time-keeping, to be susceptible to the slight change in the force of gravity produced by the motion of the moon in its apparent circuit round the earth in about 25 hours.

An explanation of the phenomenon and the method adopted for its detection is as follows:—A pendulum is dependent for its swing upon the force of gravity always tending to pull it to its centre, or zero position. In a pendulum clock small impulses are frequently given to overcome the tendency to stop at zero, and so the pendulum is maintained in vibration. If the force of gravity were to increase, the pendulum would swing faster, but for the present it may be considered that at a given place gravity is constant. It varies at different places on the earth, due to distance from the earth's centre; for instance, if a pendulum clock was set to keep accurate time in London and was transferred to a place on the Equator, it would be found to be about two minutes a day out of time, owing to change in the force of gravity between London and the Equator. Consequently, if